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Influence of spatial variability of ground motion on seismic response of bridges

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Subject review

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Influence of spatial variability of ground motion on seismic response of bridges

An approach to seismic analysis of bridges under spatially variable ground motions is presented. The phenomenon of spatial variability of earthquakes, its effects on bridge response, and differences with respect to simultaneous excitation of supports, are explained. The model of such excitation is described in detail, and procedures for generation of spatially variable ground motions are outlined. Numerical analysis methods, efficient for solving this problem, are also presented. The described methodology is applied in the seismic analysis of an arch bridge. The analysis results show that the spatial variability of ground motions has a detrimental effect on most of the analysed bridge response values.

Key words:
spatial variability of ground motion, seismic design, numerical analysis, arch bridges

Pregledni rad

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Učinak prostorne promjenjivosti potresne pobude na seizmički odziv mostova

U radu je prikazan pristup seizmičkoj analizi mostova pri prostorno promjenjivoj potresnoj pobudi. Objašnjen je fenomen prostorne promjenjivosti potresa, utjecaj na odziv mostova i razlike u odnosu na istodobnu pobudu oslonaca. Detaljno je opisan model takve pobude, s osvrtom na postupke tvorbe prostorno promjenjivih zapisa. Prikazane su i numeričke metode proračuna, učinkovite za rješenje ovoga problema. Opisana metodologija je primijenjena na seizmičku analizu lučnog mosta. Rezultati analize su pokazali nepovoljan učinak prostorne promjenjivosti pobude na odziv većine razmatranih veličina mosta.

Ključne riječi:
prostorna promjenjivost potresne pobude, seizmički proračun, numerička analiza, lučni mostovi

Übersichtsarbeit

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Einwirkung räumlich veränderlicher Erdbebenanregungen auf das seismische Verhalten von Brücken

In dieser Arbeit wird ein Ansatz zur seismischen Analyse von Brücken bei räumlich veränderlichen Erdbebenanregungen gegeben. Das Phänomen räumlicher Veränderlichkeit von Erdbeben, Einflüsse auf das Verhalten von Brücken und Unterschiede zu gleichzeitiger Belastung der Stützen ist gegeben. Ein Model solcher Einwirkungen ist ausführlich beschrieben und Verfahren zur Erstellung räumlich veränderlicher Aufzeichnungen sind erläutert. Für die Lösung dieses Problems wirksame numerische Berechnungsmethoden sind ebenfalls dargestellt. Die beschriebene Methodologie ist zur seismischen Analyse von Bogenbrücke angewandt. Resultate weisen auf ungünstige Einflüsse räumlicher Veränderlichkeit der Anregungen bei den meisten Größen untersuchter Bogenbrücken hin.

Schlüsselwörter:
räumliche veränderliche Erdbebenanregungen, seismische Berechnung, numerische Analyse, Bogenbrücken

1. Introduction

Various studies have pointed to unfavourable effects of spatial variability of earthquakes on the seismic response of large bridges [1, 2]. As distances between large bridge supports are of the order of magnitude of the earthquake wavelength (finite propagation velocity from the source toward the surface), it is clear that seismic excitation cannot simultaneously affect all bridge supports. In addition, foundation conditions may not be the same along the entire length of the bridge. The simplest example of an unfavourable seismic response of bridges due to such excitation is the superstructure collapse due to relative displacements of its supports, which is especially pronounced in the case of superstructures constructed as a series of simply supported beams.

First studies focusing on spatial variability of ground motion were published in 1960's [3]. They covered only the effect of seismic waves delay on the more remote bridge supports (in brief, seismic wave-passage effect). After the arrays of densely distributed measurement stations were set up in various parts of the world (such as SMART-1 on Taiwan, El Centro Differential array, EPRI Parkfield, Hillister, Coalinga, Pinyon Flat in the USA, Chiba in Japan, and Thessaloniki and Argostoli in Greece [4]), where information on ground motion during earthquakes is recorded, the investigations have been extended to some other phenomena that cause spatial variability of ground motions. In addition to the already mentioned wave-passage effect, these phenomena include coherency loss effects, wave attenuation effects, and effect of local site conditions (Figure 1).

The wave-passage effect accounts for the time lag of seismic excitation, i.e. the time needed for the wave to travel from one support to another. This effect is described by the apparent propagation velocity, which represents the speed of wave propagation along the ground surface. The apparent velocity may be determined through analysis of data recorded during an earthquake, and it is mostly assumed with constant value (the dependence on frequency is rarely taken into account). Further information on the apparent velocity of wave propagation and its determination may be found in [7-9].

The loss of coherency occurs due to the reflection and refraction of seismic waves in heterogeneous soil medium, and the superposition of waves coming from the seismic source. It is described via the coherency function that decreases with the distance and frequency, and may be determined analytically, empirically, or semi-empirically. If the coherency function is considered using the random process theory, it represents the ratio of cross-power spectrum density of two records at different locations to the square root of the product of two corresponding auto-power spectrum densities. In general terms, the coherency function is complex-valued, as described in greater detail in Section 2.2.

The wave attenuation effect accounts for the gradual decay of seismic wave amplitudes due to the geometric spreading and energy dissipation on the ground medium. However, this effect is not usually taken into account in the models of spatial variability of ground motion, as it exerts no significant effects on structures [6].

The effect of local site conditions comprises differences in local soil conditions at different bridge supports. These differences affect the frequency range and amplitudes of seismic waves, and so amplitudes increase when seismic waves pass through some soils, while they decrease in some other soils. The effect of local site conditions can significantly influence structures with supports situated in different types of soil (e.g., in case of bridges over rivers and bays). This effect is usually modelled in a simplified way, i.e. using only different power spectral density functions or elastic pseudo-acceleration spectra for soil under the supports. A more detailed approach to modelling of local site conditions may be found in [8, 10-12].

Due to unprecedented advancements in knowledge, technology and computer efficiency in the 1990's, the research of spatial variability of ground motion has since that time become more extensive, and numerical models have gained in complexity and sophistication. A detailed review of research on response of bridges may be found in [8, 13], and so reference to a particular study will be made in this paper only when relevant. In fact, although many studies have so far been made on the response of various bridge types, most of them focus on the behaviour of girder bridges, which are most frequently encountered in practice. Some significant studies on the response of these bridges may be found in [1, 2, 5, 14-27], while the research on the response of suspension bridges and cable-stayed bridges, with regard to spatial variability of ground motion, may be found in [28-40]. It can generally be stated that not much research has been made regarding the effect of spatial variability of seismic motion on the response of arch bridges. Some of this research is presented in [13, 33, 41-47].

Many studies have shown that it is very difficult to determine the effect of an individual spatial variability parameter on the response

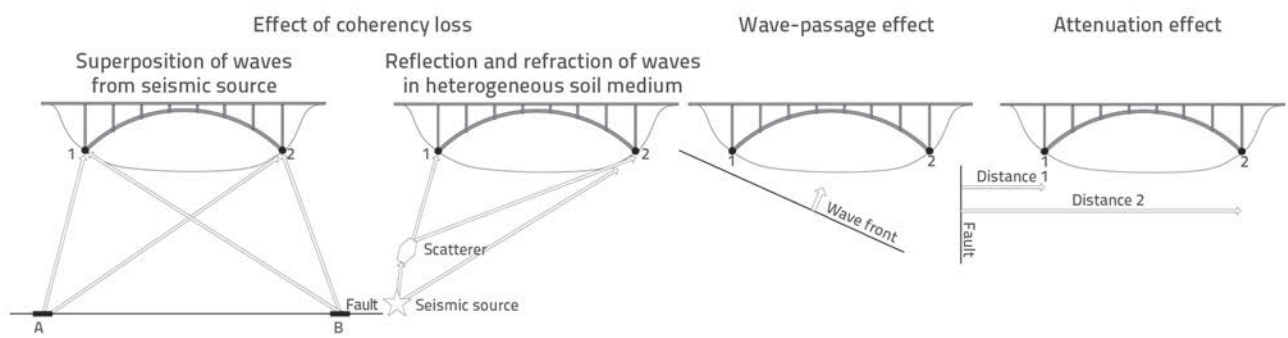

Figure 1. Principal forms of spatial variability of ground motion [4-7]

of a structure because, in practice, there are many possibilities regarding the bridge layout, materials, soil properties, and design criteria. If these possibilities are combined with possible scenarios of seismic excitation, which is by itself a random and poorly predictable process, the complexity of the issue becomes even more pronounced. Studies cover only a small portion of possible scenarios and, as they cannot be deemed systematic, it is very difficult to use them as basis for making general conclusions. Even in the cases of very similar structures and excitations, the results of analyses are often quite contradictory (see, for instance, research conducted in [17, 19]). On top of that, researchers cannot agree on whether the spatial variability of ground motions has beneficial or detrimental effect on the response of structures. In fact, although it is clear that the spatial variability causes different response than uniform support excitation, it is not easy to predict whether the response will actually decrease or increase. Due to relative displacements of supports, spatially variable excitations induce pseudo-static response component and additionally, the dynamic component changes as different vibration modes are excited than those due to uniform excitation. This all shows that the effect of spatially variable excitation is a highly complex phenomenon, dependent on many parameters that describe excitation, but also the structure. Of course, this does not mean that the phenomenon of spatial variability of ground motion should not be considered. On the contrary, further research, oriented toward the engineering practice, is proposed with regard to its effect on bridges of various layouts, all aimed at expanding the current database on this phenomenon [48].

Experimental studies on spatial variability are very rare because a system involving multiple shaking tables is needed to test the response and so, according to available information, such tables exist only in several university and research centres worldwide (e.g. in the USA: University of Nevada - Reno, University at Buffalo, SUNY's Structural Engineering and Earthquake Simulation Laboratory; in China: Chongqing Communications Research and Design Institute, and in Italy: ISMES, Bergamo). The information on these experimental studies may be found in [40, 49-53].

The objective of this paper is to provide an overview of the state-of-the-art in this area, with a detailed bibliography, and to present a typical example of numerical analysis of a bridge, which is expected to be of interest for engineering practice.

2. Approach to seismic analysis for spatial variability of ground motion

2.1. Formulation of equation of motion

If a structure is excited by spatially variable ground motions, individual foundations do not move in the same way because the distance between them becomes a time dependent variable. The formulation of the system equations of motion differs from formulation due to the uniform excitation (which corresponds to the absolutely stiff soil), as the static part caused by relative displacement of supports must be added to dynamic response. The differential equation describing the system motion, with n degrees of freedom for the structure, and m degrees of freedom for the supports, can be written as follows:

$$\begin{bmatrix} \mathbf{m} & \mathbf{m_g} \\ \mathbf{m_g}^T & \mathbf{m_{gg}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}^t \\ \ddot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c_g} \\ \mathbf{c_g}^T & \mathbf{c_{gg}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}^t \\ \dot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k_g} \\ \mathbf{k_g}^T & \mathbf{k_{gg}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^t \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p_g}(t) \end{Bmatrix} \quad (1)$$

where **m**, **c** and **k** are n-th order matrices of the mass, damping, and stiffness, $\mathbf{m}_{gg}$, $\mathbf{c}_{gg}$ and $\mathbf{k}_{gg}$ are m-th order matrices of support models, while $\mathbf{m}_g$, $\mathbf{c}_g$ and $\mathbf{k}_g$ are $n \times m$ matrices due to interaction between the models of the structure and supports. The displacement vector consists of two parts: one part contains degrees of freedom for the structure $\mathbf{u}^t = [u_1^t, \ldots, u_n^t]^T$ and the other part is formed of support displacements $\mathbf{u}_g = [u_{g1}, \ldots, u_{gm}]^T$. Displacements belonging to the structure, $\mathbf{u}^t$, can be divided into displacements due to static application of the prescribed support displacement, $\mathbf{u}^s$ (which change slowly over time – quasi-static part) and into dynamic displacements **u** that can only be determined by dynamic analysis:

$$\mathbf{u}^t = \mathbf{u}^s + \mathbf{u} \quad (2)$$

The relationship between quasi-static displacements and support displacements can be established as follows:

$$\begin{bmatrix} \mathbf{k} & \mathbf{k_g} \\ \mathbf{k_g}^T & \mathbf{k_{gg}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^s \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p_g}^s \end{Bmatrix} \quad (3)$$

that corresponds to equation (1) but without the inertial and damping terms. In exp. (3) $\mathbf{p}_g^s$ are the forces on supports exerting static displacements $\mathbf{u}_g$, which still vary over time.

It may be seen in the first row of equation (3) that $\mathbf{k}\mathbf{u}^s + \mathbf{k}_g\mathbf{u}_g = \mathbf{0}$, so that the relationship between static displacements $\mathbf{u}^s$ and support displacements $\mathbf{u}_g$ can be established as shown below:

$$\mathbf{u}^s = -\mathbf{k}^{-1}\mathbf{k_g}\mathbf{u_g} = \ell\, \mathbf{u_g} \quad (4)$$

where ℓ is called the influence matrix as it defines the influence of support displacements on the displacements of the structure. By inserting (2) and (4) in the first row of equation (1), we obtain the following equation of motion:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -(\mathbf{m}\ell + \mathbf{m_g})\ddot{\mathbf{u}}_g(t) - (\mathbf{c}\ell + \mathbf{c_g})\dot{\mathbf{u}}_g(t) \quad (5)$$

where the second right hand side term is often neglected as damping forces are usually much smaller compared to inertial forces.

2.2. Model of spatial variability of ground motion

The spatial variability of ground motion is often described in a probabilistic manner using the space-time random field of ground motion, with data obtained from the series of dense instrument arrays. In this way, the model of spatial variability of ground motion can be expressed through cross-power spectral density of the motions $S(\xi_{jk},\omega)$ as:

$$S(\xi_{jk},\omega) = S(\omega)\cdot\gamma_{jk}(\xi_{jk},\omega) \qquad (6)$$

where $S(\omega)$ is the power spectral density function of the motions, ξ_{jk} is the distance of supports j and k, and $\gamma_{jk}(\xi_{jk},\omega)$ is the complex coherency.

This model is directly used in the analyses based on the theory of random vibrations, or indirectly for the generation of ground motions at different supports, that are needed for the time-history methods [8].

Out of the proposed power spectral density functions, the expansion of Kanai-Tajimi spectra [55], developed by Clough and Penzien [56], is usually selected for practical applications. The original function represents the spectral density of ground motion with the constant spectral density S_0 at the bedrock level, in the entire frequency range (the so called white noise). The function is then filtered through soil layers characterized by the single degree of freedom system of natural frequency ζ_g and damping ω_g. Clough and Penzien used additional filter with parameters ω_f and ζ_f in order to avoid numerical difficulties in the frequency area ω close to zero, and so that the extended power spectral density function of ground acceleration is:

$$S(\omega) = S_0\cdot\frac{1+4\zeta_g^{\,2}(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2+4\zeta_g^{\,2}(\omega/\omega_g)^2}\cdot\frac{(\omega/\omega_f)^4}{\left[1-(\omega/\omega_f)^2\right]^2+4\zeta_f^{\,2}(\omega/\omega_f)^2} \qquad (7)$$

The values of parameters S_0, S_0, ω_g, ζ_g, ω_f and ζ_f may be found in [33].

The complex coherency $\gamma_{jk}(\xi_{jk},\omega)$ contains the amplitude and phase parts for each frequency range:

$$\gamma_{jk}(\xi_{jk},\omega) = \left|\gamma_{jk}(\xi_{jk},\omega)\right|\cdot e^{i\theta_{jk}(\xi_{jk},\omega)}, \qquad i=\sqrt{-1},$$
$$\theta_{jk}(\xi_{jk},\omega) = -\frac{\omega\cdot(\overline{V_{\mathrm{app}}}\cdot\overline{\xi_{jk}})}{\left|\overline{V_{\mathrm{app}}}\right|^2} = -\frac{\omega\cdot\xi_{jk}^{\,L}}{V_{\mathrm{app}}} \qquad (8)$$

The amplitude part (real member representing the loss of coherency $|\gamma_{jk}(\xi_{jk},\omega)|$ is most often described by means of the term lagged coherency, which is the measure of linear statistic dependence between two processes. If its value is zero then the processes are completely independent from one another and, if the value is one, the processes are perfectly linearly dependent. The phase part $e^{i\theta_{jk}(\xi jk,\omega)}$ represents the wave-passage effect or, more precisely, the time lag in the arrival of seismic waves to the supports j and k, where $\theta_{jk}(\xi_{jk},\omega)$ is the phase angle dependent on the distance of supports and frequency. Finally, $\xi_{jk}^{\,L}$ is the distance between supports j and k (ξ_{jk}) projected in the direction of wave propagation, while V_{app} is the apparent velocity of wave propagation [57].

Most coherency functions obtained empirically and semi-empirically are determined by the analysis of data recorded in the SMART-1 array in Taiwan [20]. Although many coherency functions have been proposed, the one presented by Luco and Wong [58] has been used in most studies:

$$\left|\gamma_{jk}(\xi_{jk},\omega)\right| = e^{-\alpha^2\omega^2\xi_{jk}^{\,2}} \qquad (9)$$

The term α controls the exponential drop of the coherency function with an increase of distance and frequency. The greater the value of this parameter, the greater is the loss of coherency. Luco and Wong propose the value of $\alpha \approx (2\text{-}3)\times10^{-4}$ s/m.

Harichandran and Vanmarcke developed a coherency function that is also popular among the researchers [59]. They estimated lagged coherencies from the data recorded at SMART-1 stations during four earthquakes. They proposed the following expression:

$$\left|\gamma_{jk}(\xi_{jk},\omega)\right| = Ae^{\left[-2\xi_{jk}/(\alpha\theta_\omega)(1-A+\alpha A)\right]}+(1-A)e^{\left[-2\xi_{jk}/\theta_\omega(1-A+\alpha A)\right]};$$
$$\theta_\omega = k[1+(\omega/\omega_0)^b]^{-0,5} \qquad (10)$$

where θ_ω is the frequency-dependent spatial scale of fluctuation, while A, α, k, ω_0, and b are empirical parameters, for the firm soil [33] amounting to: $A = 0.626$, $\alpha = 0.022$, $k = 19700$ m, $\omega_0 = 12.69$ rad/s and $b = 3.47$.

The selection of coherency function greatly influences the seismic response of structures subjected to spatially variable excitation. For the quasi-static part of response, this is especially pronounced in the low frequency range. It was established that the function developed by Harichandran and Vanmarcke is less appropriate for "rock" sites (it was obtained from empirical data recorded at the SMART-1 array, located on alluvial soil), and that it is only partly correlated at lower frequencies. The function developed by Luco and Wong is completely correlated in that range. Physically, the coherency model should tend to unity as frequency and separation distance tend to zero [8].

2.3. Numerical analysis procedures

The following numerical methods are used in the study of spatial variability of ground motions: random vibration methods, response spectrum methods and deterministic methods.

In the random vibration method, the seismic excitation is described through the power spectral density, while the spatial variability of ground motions is described by introducing the coherency loss function. Stochastic methods may be used quite efficiently to take into account all significant earthquake spatial variability phenomena as random occurrences. However, the methods are limited to linear analyses, and the practical use is complex as calculations result in statistical measures of response only.

The response spectrum specified in most international seismic-design standards cannot directly be applied in the analysis of spatial variability of ground motions. Response spectrum methods for spatially variable excitation are based on the random vibration theory, but the response spectrum, rather than the power spectral density function, is used for defining the excitation. An additional advantage is that the non-stationarity of ground motion is included in the spectrum definition. Der

Kiureghian and Neuenhofer [5] developed a basic response spectrum method for the spatially variable seismic excitation (abbreviated as MSRS). The method takes into account the effect of correlation between motion of supports and natural modes of vibration of the structure. Spectral density and coherency functions were used to account for the wave-passage, loss of coherency, and local-site effects. The method gives the mean value of peak structural response. Konakli and Der Kiureghian proposed a generalized and extended basic spectrum response method (MSRS) by Der Kiureghian and Neuenhofer, and developed a computer code for implementation of the method. The initial formulation of the MSRS method takes into account only those response values that can be expressed through linear combination of displacements in the direction of degrees of freedom of the structure. The generalized formulation may also contain the response values that include one or more degrees of freedom of supports, and it takes into account the quasi-static contribution of higher vibration modes.

The most general procedure of dynamic response calculation is based on deterministic time-history methods, where the equations of motion are solved by time stepping methods. As there are many procedures for numerical solution of the equations of motion, the reader is advised to consult extensive literature, such as [54, 60, 61]. Time stepping methods may also be used to solve nonlinear problems and therefore, they are often used in practice, although they may be numerically demanding and time consuming. As they are currently the most powerful tool for solving dynamic equations, some attention will be given in the paper to the ground motion simulation techniques that are used in the implementation of these methods.

Numerical models of the system may vary in complexity. The most general model of the bridge is the three-dimensional nonlinear model that is solved by the time stepping methods with time-histories applied in three orthogonal directions (analyses in three orthogonal directions cannot be examined independently, as superposition principle does not apply in the nonlinear range). When the soil-structure interaction effects (kinematic and inertial effects) are significant, then they have to be taken into account. Various simplified and advanced approaches to this problem are now used, that can be divided into direct approaches and the sub-structuring approaches where the structure and soil-foundation system are separated and studied accordingly. When the direct approach to the problem is applied, the entire soil-structure system is modelled as one unit. The seismic excitation is specified at the bedrock level, and the complex propagation of waves through various soil layers is simulated numerically. The 2D bridge model and the 2D soil model may be used or, less often, 3D model of soil and bridge. According to available data, only several studies based on the direct approach including the effect of spatial variability of ground motions, have so far been made. For instance, Yang et al. [62] investigated the response of a girder bridge with piles and the surrounding soil, using the 2D model created in the OpenSees program developed at Berkeley. The sub-structuring approach to the soil-structure interaction problem is often applied in practice. Here, the structure and soil-foundation system are considered separately and the main numerical model contains only structural elements, while the soil-structure interaction is taken into account via support conditions with characteristics based on detailed geotechnical analyses. One such procedure is proposed in [10-11]. In brief, it is first necessary to generate time-histories using a spatial variability model that takes into account the effects of wave passage, loss of coherency of seismic waves, and local-site conditions. Then these time-histories should be modified in the frequency domain so as to take into account the kinematic interaction between the soil and the foundation. This defines the input seismic action at the foundation level. The procedure continues by defining properties of bridge supports modelled by the spring-dashpot systems, whose dynamic impedance matrices are derived for all necessary vibration modes. Then dynamic analysis may be performed with a relatively simple numerical model of the bridge using any commercial finite-element software.

However, regardless of the approach used, attempts should always be made to create a model of reasonable complexity, which should be in accordance with the reliability of input data.

2.4. Generation of spatially variable seismic ground motions

Seismic ground motions can generally be obtained by recording ground shaking during earthquakes in a favourable array of closely distributed stations, by modelling the seismic source and wave propagation through an elastic soil medium, and by simulation of spatial variability of ground motions based on probabilistic models.

The response of structures to a spatially variable ground motions cannot be analysed using the recorded accelerograms only, as we would have to have at our disposal a great number of simultaneously recorded time-histories made at different distances, in different soils, of different magnitudes, etc., for all possible bridge layouts. As such a seismic record database obviously does not exist, we have to additionally make use of one of the record generation methods.

The development of algorithms for generation of random processes has enabled advancements in the study of spatial variability of ground motions. Generally, seismic time-histories are generated using a specific probabilistic model with empirical data obtained by analysis of records from the array of closely distributed stations. The power spectral density function or the response spectrum is used in that procedure, combined with function of complex coherency. The generated samples must accurately describe probabilistic properties of appropriate random processes, fields, or waves that can be either stationary or non-stationary, homogeneous or non-homogeneous, one-dimensional or multidimensional, Gaussian or non-Gaussian, uni-variate or multi-variate [20]. Many methods for the generation of such samples have so far been proposed. One of the most frequently used methods is the so called spectral representation method. This procedure was used by Deodatis [63] to elaborate in detail the theory, simulation algorithm, and iteration scheme for generation of the acceleration time-histories as multi-variate, non-stationary stochastic processes. This procedure is known as the unconditional simulation. The drawback of the method lies in the use of the power spectral density function, rather than the response spectrum, which is very widely used in the engineering practice. That is why Deodatis proposed an additional iteration procedure in which simulated time-histories are adjusted to be compatible with the prescribed response spectrum until an appropriate matching is achieved, as described below.

Consider that the ground motions are to be generated at n points on the ground surface as non-stationary stochastic vector processes with n variables and with the uniform modulating function independent of the frequency ω. Generally, points correspond to different local soil conditions, and consequently, a different target response spectra are assigned to these points: $RSA_j(\omega)$; $j = 1,\dots,n$. Complex coherency functions $\gamma_{jk}(\omega)$; $j,k = 1,\dots,n$; $j \neq k$ are prescribed between the pairs of points (dependence on distance ξ_{jk} will be omitted as these values are directly inserted in the calculation of the function: (ω) is inserted instead of (ξ_{jk}, ω)). Modulating functions $A_j(t)$; $j = 1,\dots,n$ are assigned to each point. The procedure starts by generation of the ergodic stationary time-histories as stochastic processes with n variables, compatible with the specified model for spatial variability of ground motions. The model can be described by the cross-spectral density matrix of ground motions $\mathbf{S}^0(\omega)$:

$$\mathbf{S}^0(\omega) = \begin{bmatrix} S_1(\omega) & \sqrt{S_1(\omega)S_2(\omega)}\,\gamma_{12}(\omega) & \cdots & \sqrt{S_1(\omega)S_n(\omega)}\,\gamma_{1n}(\omega) \\ \sqrt{S_2(\omega)S_1(\omega)}\,\gamma_{21}(\omega) & S_2(\omega) & \cdots & \sqrt{S_2(\omega)S_n(\omega)}\,\gamma_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{S_n(\omega)S_1(\omega)}\,\gamma_{n1}(\omega) & \sqrt{S_n(\omega)S_2(\omega)}\,\gamma_{n2}(\omega) & \cdots & S_n(\omega) \end{bmatrix} \quad (11)$$

where diagonal members $S_{jj}^0(\omega) = S_j(\omega)$ represent power spectral densities of time-histories at each support j, while the off-diagonal elements $S_{jk}^0(\omega) = \sqrt{S_j(\omega)S_k(\omega)} \cdot \gamma_{jk}(\omega)$ are the corresponding cross-spectral densities between the supports j and k. Since the compatibility of the generated samples with the target response spectra rather than with the power spectral density function is required, the function $S_j(\omega)$ at the beginning of the procedure (in the first iteration) may be assumed almost arbitrarily.

The matrix $\mathbf{S}^0(\omega)$ should then be decomposed using Cholesky's method to the product:

$$\mathbf{S}^0(\omega) = \mathbf{H}(\omega)\mathbf{H}^{T^*}(\omega) \quad (12)$$

where T* denotes the transpose of the complexly conjugated matrix. $\mathbf{H}(\omega)$ is the lower triangular matrix whose diagonal elements are real and positive functions of ω, while off-diagonal elements are generally complex functions of ω.

After decomposition of the matrix $\mathbf{S}^0(\omega)$ according to expression (12), the non-stationary stochastic vector process can be simulated via the product of the modulating function $A_j(t)$ (by which the non-stationarity of the process is introduced) and the stationary process $g_j(t)$, in the following way:

$$f_j(t) = A_j(t) \cdot g_j(t) = A_j(t) \cdot \underbrace{2\sum_{m=1}^{n}\sum_{\ell=1}^{N} \left|H_{jm}(\omega_\ell)\right| \sqrt{\Delta\omega} \cos\left[\omega_\ell t - \theta_{jm}(\omega_\ell) + \phi_{m\ell}\right]}_{g_j(t)}; \quad j = 1,\dots,n \quad (13)$$

In this expression $\Delta\omega = \omega_u / N$ is the frequency step, $\omega_\ell = \ell \cdot \Delta\omega$ for $\ell = 1,\dots,N$ are discrete values of frequencies, and ω_u is the upper cut-off frequency beyond which cross-spectral density matrix elements may be assumed equal to zero for any time instant t. The selection criterion for the value ω_u may be found in [64]. Furthermore, $\phi_{m\ell}$ ($m = 1,\dots,n$ i $\ell = 1,\dots,N$) are independent random phase angles, uniformly distributed over the interval [0, 2π], while $\theta_{jm}(\omega_\ell)$ is the phase angle defined through members of the matrix $\mathbf{H}(\omega)$ according to the expression:

$$\theta_{jm}(\omega_\ell) = \tan^{-1}\left(\frac{\mathrm{Im}\left[H_{jm}(\omega_\ell)\right]}{\mathrm{Re}\left[H_{jm}(\omega_\ell)\right]}\right) \quad (14)$$

It should also be noted that the generation of stationary and ergodic stochastic vector processes can efficiently be carried out using fast Fourier transforms.

After the performed simulations, the elastic response spectrum is determined for each generated motion and compared with the target spectrum. If necessary, the Fourier amplitude is adjusted by adapting the power spectral density function using the product of the ordinate of the initial spectral density function in this iteration, and the square of the ratio of the target response spectrum to the response spectrum of the generated motion.

Then stationary processes are once again generated and multiplied by modulating functions in order to obtain non-stationary processes. The procedure is repeated until satisfactory compatibility is obtained between response spectra of generated time-histories and the target response spectra. Only several iterations are usually needed for a sufficiently accurate convergence. The specific criterion is not used as the change of one frequency component of time-history also influences values of response spectra in other components, and so the convergence criterion of the iteration procedure cannot easily be determined, and it is not expected that the procedure will perfectly converge at all frequencies.

The conditional simulation method developed by Vanmarcke et al. [65] can also be used for generation of spatially variable time-histories. In this generation procedure, a recorded time-history is used in order to generate ground motions at other supports based on a spatial variability model. The input parameters are power spectral density functions and coherency functions for several stationary time windows that do not overlap. The procedure does not take into account possible time gap between seismic waves, which does not create any problems in practice as this effect can simply be specified in a computer program using differences in the phase of time-histories. The method can be used for the generation of time-histories needed for seismic analysis of new structures, or for simulating a record on the location of a collapsed or damaged structure where the accelerograph did not exist, but the ground shaking was recorded at nearby location. The procedure uses a linear-prediction method for the generation of statistically independent random processes in a specific frequency range using fast Fourier transforms. The implementation of the conditional simulation procedure will not be explained in detail in this paper. However, the reader interested in the topic is referred to [65] and the existing Fortran software code SIMQKE-II developed by the same authors [66]. The selection of the simulation technique depends on the specific problem encountered, and the objective is to ensure adequate match between generated ground motion properties and the specified criteria.

2.5. Review of European standard for seismic design of bridges [67]

According to the standard for seismic design of bridges, the influence of spatial variability of ground motion shall be considered for bridges with continuous superstructure if soil properties along the bridge are variable and/or if soil properties along the bridge are mostly uniform, but the length of the continuous superstructure exceeds an appropriate limit length L_{lim}. The model describing spatial variability should take into account the influence of the wave-passage, loss of coherency, and differences in mechanical properties of the soil along the bridge. Besides the time-history analysis using seismic motions, a simplified method may also be used. It consists of the combination of the most unfavourable dynamic response and the most unfavourable effects obtained by quasi-static calculations, using the well-known SRSS rule. Thereby, the dynamic response is estimated using single input seismic action for the entire structure, corresponding to the least favourable ground type underneath the bridge supports, while the quasi-static part is defined by imposing appropriate sets of displacements on the relevant support foundations or on the soil end of the relevant spring. In the first case, relative displacements with the same sign are applied and in the second case displacements at adjacent piers are specified in the opposite direction. The informative annex D contains guidelines for the generation of seismic motions based on the random process method using the power spectra consistent with the elastic response spectrum at supports and coherency function according to [6, 58]. Appropriate analysis methods of structures subjected to spatially variable seismic excitation are presented: linear random vibrations analysis, time-history analysis with the generated seismic motions, and use of response spectrum method for the spatially variable excitation according to [5].
A simplified calculation method for spatially variable excitations, proposed in the standard [67], is evaluated in full detail in [26]. Based on an extensive analysis of 27 structures, it was concluded that there are special cases in which provisions given in the standard may be applied easily and safely, while in some cases the use of the simplified procedure proposed in the standard is explicitly discouraged as the obtained results are not on the safe side. In cases when provisions contained in the standard did not produce satisfactory results, alternative solutions are proposed including corrections of some expressions given in the standard.
It is shown in [13] that the use of simplified procedures for consideration of spatially variable seismic excitation is not recommended for RC deck arch, as the comparison between results obtained by the simplified method and those obtained by time-history methods has pointed to significant deviations, with the considerably underestimated response of structures in almost all cases.

3. Numerical analysis of arch bridge accounting for spatial variability of ground motion

The phenomenon of spatial variability of seismic excitation and its effect on the structural response is explained in previous sections 1 and 2. Numerical procedures used in the analysis of such problems are presented, and some methods for generation of spatially variable seismic time-histories needed for the analysis using time stepping methods are described. In this section, a simple example of an arch bridge will be used to explain the seismic analysis of a structure considering the spatial variability of earthquake.

3.1. Numerical model of the bridge

The arch bridge is shaped in form of a catenary . The arch span is 100 m, the arch rise is f_L=20 m, and springing to crown load ratio is m=3. Cross-sectional dimensions of the superstructure, piers, and arch are taken from [72]. The arch cross section is a two-cell box 10,0 m wide and 2,0 m deep with 30 cm thick webs and 45 cm thick chords , of constant dimensions along the arch. Piers are also of box-type cross section and they measure 3.0 x 1.5 m,

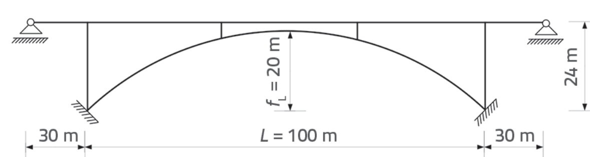

Figure 2. Numerical model of the bridge

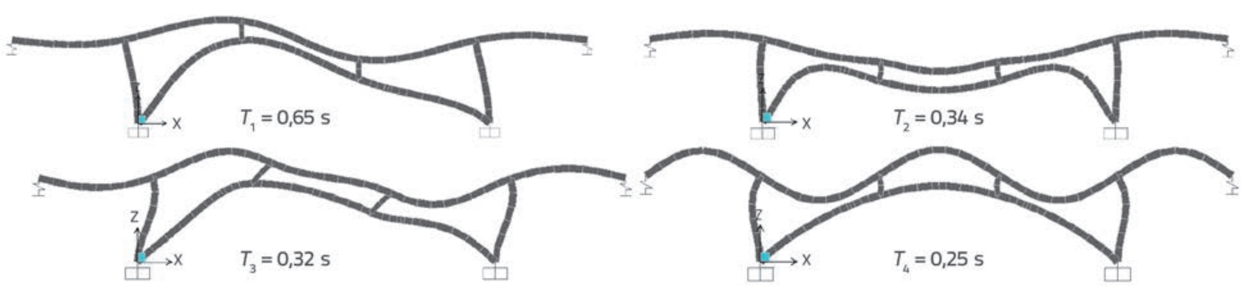

Figure 3. First four modes of bridge vibration

with 30 cm thick walls for all piers. Portal pier dimensions are 4.0 x 2.0 m, with 50 cm thick walls. The superstructure cross section is a reinforced-concrete box, with 35 cm thick top plate 20 m wide. Other dimensions are: box depth 2.0 m, box bottom plate: 10 m wide and 30 cm thick, and webs: 60 cm thick. The arch and superstructure are made of concrete class C45/55, while class C35/45 is used for piers [71]. Transverse diaphragms are placed inside the arch under the spandrel piers. Their width is equal to the width of the spandrel piers that they support.

A two-dimensional numerical model was developed (Figure 2) with arch springings modelled as fixed supports (the bridge is founded on solid rock). The calculation was performed using the software SAP2000 Nonlinear, ver. 14.2.4 [73].

The geometrical nonlinearity was to be taken into account during definition of the numerical model. However, the comparison of responses of geometrically linear and nonlinear models did not reveal significant deviations (the difference is less than 10 %). The reduction in seismic force because of energy dissipation through concrete cracking and considerable reinforcement yielding, was not taken into account as the arch response is (mostly) not ductile due to considerable arch thrust [74]. In other words, only the losses in elastic region were accounted for.

Because of demanding calculations and numerical difficulties that may occur when solving eigen-values problem, the calculation of vibration modes was based on the Ritz vector subspace. In fact, during dynamic analysis of structures under seismic excitation defined using displacement time-histories that excite higher vibration modes, a greater number of vectors must be defined to achieve almost full participation of the model mass in the translational direction. 45 Ritz vectors were needed for this purpose. First four bridge vibration modes are shown in Figure 3.

3.2. Actions and analysis method

Actions considered in seismic design comprised permanent actions (self-weight and bridge equipment as required for the motorway) and the seismic action in the direction of the bridge axis, specified using ground displacement time-histories.

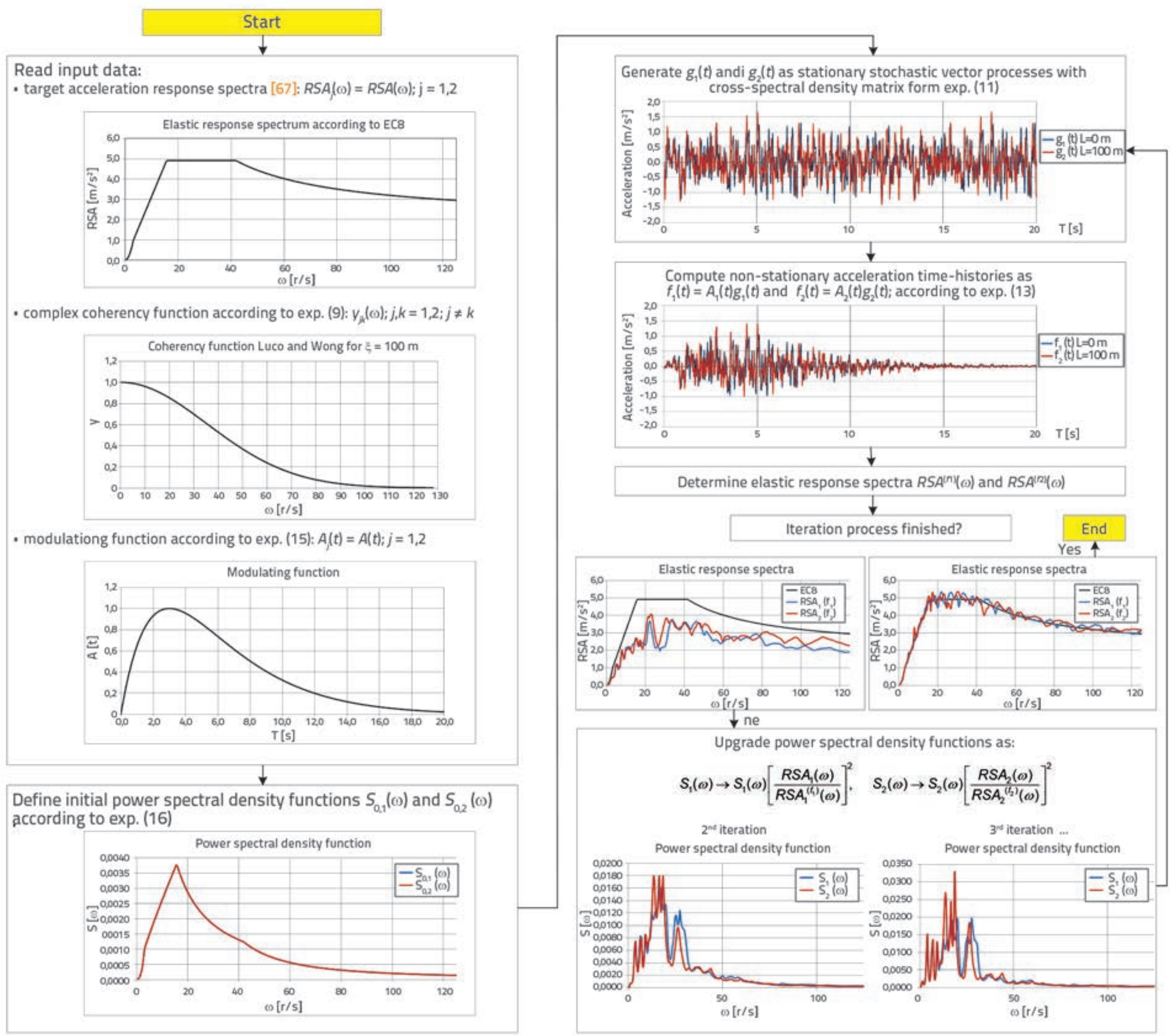

Figure 4. Scheme of iteration procedure for generating seismic motions compatible with target response spectrum

The following types of seismic excitation were analysed:
- Case 1 (UNIFORM): uniform (simultaneous) seismic excitation at supports,
- Case 2 (COH): loss of coherency (infinite velocity of seismic waves),
- Case 3 (WP): wave-passage effect at apparent velocity V_{app}=2500 m/s (coherent seismic waves)
- Case 4 (COH+WP): simultaneous wave-passage WP + loss of coherency COH.

Seismic motions were generated using the unconditional simulation method based on a probabilistic model [63] and the conditional simulation method with the recorded time-history [65], as described in Section 2.4. Thirteen ground motion sets were defined for the arch springings using the unconditional simulation for generation of seismic motions compatible with the prescribed response spectrum. The procedure was programmed using the Wolfram Mathematica software [75]. The generation was performed according to the iteration scheme shown in Figure 4. Less than five iterations were needed for most time-histories to achieve good compatibility between their elastic response spectra and the target elastic response spectrum.
The following parameters were selected during generation of seismic motions at different bridge supports founded on rock: V_{app} = 2500 m/s and the coherency function according to [58] with a low value of the coherency drop parameter α = 2,0×10^{-4} [8]. The elastic response spectrum for horizontal seismic action from the valid European standard [68] is defined by the type of spectrum 1 (M_S > 5,5), damping ζ = 5 %, peak ground acceleration a_{gR} = 0,20g, design ground acceleration $a_g = g_I \cdot a_{gR}$ = 0,20 g, and ground type A (S=1,00; T_B =0,15 s; T_C=0,40 s, T_D=2,0 s).
In addition to these assumptions, it was also necessary to define the modulating function (so as to obtain non-stationary time-histories from the generated stationary ones), then the initial power spectral density function, and the required number of steps. The total duration of motion is T = 20 s, with the time step of Δt = 0,01 s (the number of data is $T/\Delta t$ = 2000).
The upper cut-off frequency was assumed to be ω_u = 128 rad/s. The frequency sampling was made for the total number of N = 256 frequencies spaced at $\Delta\omega = \omega_u/N$ = 0,5 rad/s. The modulating function was determined according to [76] as follows:

$$A_j(\omega,t) = A_j(t) = a_1 \cdot t \cdot e^{(-a_2 t)} \quad (15)$$

where a_1 and a_2 are parameters dependent on the earthquake magnitude and epicentral distance and their values, amounting to a_1 = 0,906 and a_2 = 1/3, respectively, are assumed in this paper according to [20].
When selecting the length of the time-history, modulating function and its defining parameters, one should bear in mind that the valid European standard for seismic design [68] defines in section 3.2.3.1.2(3), which is related to generation of artificial accelerograms, the smallest duration of the stationary part of accelerogram of 10 s, if the data for a particular location are not available. Although various modulating functions could be used for generation of time-histories in accordance with the standard, the mentioned requirements can easily be met by using the modulating function that has the constant value in the central part of duration (Figure 5). The frequently used functions are trapezoidal function [77], function developed by Amin and Ang [78], and the function according to Jennings et al.[79].

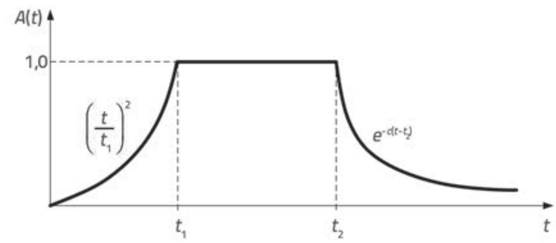

Figure 5. Modulating function [79]

In order to reduce the number of iterations needed to obtain satisfactory matching between the elastic response spectra of generated motions and the prescribed response spectrum, the function dependent on the target response spectrum's ordinates according to [12] was selected in the paper for the initial power spectral density function:

$$S(\omega) = -\frac{\zeta}{\pi\omega} \cdot \frac{RSA^2(\omega)}{\ln(-\frac{\pi}{\omega T}\ln p)} \quad (16)$$

In the expression (16) ζ is the damping coefficient, RSA denotes ordinates of the prescribed response spectrum, T is the total duration of the time-histories, and p is the probabilistic coefficient $p \geq$ 0,85 (the value of p = 0,85 is adopted in the paper).
Because of the great number of generated samples (13×2), Figure 6 shows only one set of acceleration time-histories and the corresponding displacements (obtained by double integration of analysed acceleration time-histories). Elastic response spectra are shown in Figure 7 for all generated time-histories. Black colour denotes the mean value of all spectra, while the blue colour denotes the prescribed elastic response spectrum multiplied by the factor 0.9.

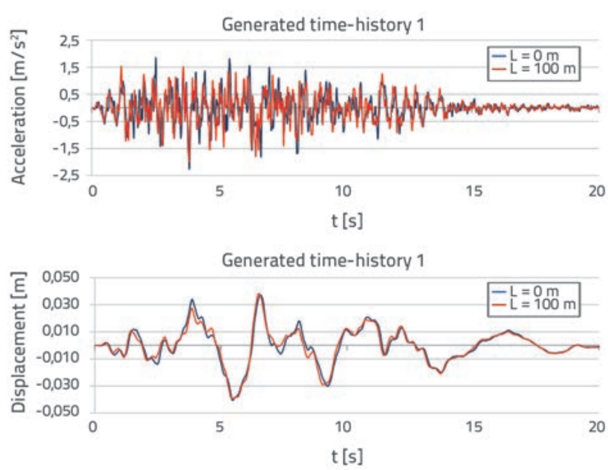

Figure 6. Generated ground acceleration time-histories and corresponding displacements obtained by the unconditional simulation procedure

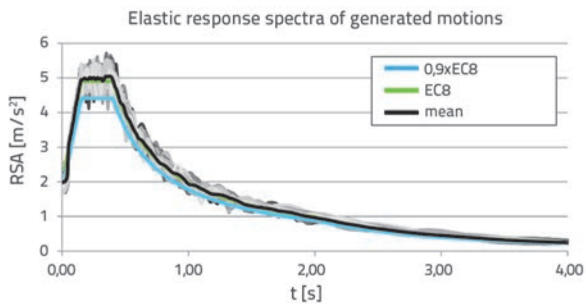

Figure 7. Elastic response spectra for seismic motions generated by unconditional simulation

Seven time-history sets were generated by means of the conditional simulation procedure for various bridge supports using the records registered on solid rock. The records were selected from the European Strong-Motion Database ESD [69] using the software program REXEL, ver. 3.5 [70]. The mean value of ordinates of the elastic response spectra of individual records is in accordance with the spectrum specified in the valid European standard. Or, more precisely, the ordinate of an average response spectrum for a group of earthquakes must exceed 90 % of the elastic spectrum specified, in the period range from $0{,}2T_1$ to $1{,}5T_1$ [67, 68].

The generation procedure was carried out using the existing Fortran programming code SIMQKE-II [66] to which a new coherency function was added.

Different colours are used in Figure 8 to present elastic response spectra for individual seismic records. Black colour denotes the mean value of all spectra, and the blue one stands for the prescribed elastic spectrum multiplied by factor 0.9. Spectral curves are determined for all time-histories using the software program Seismospect [80]. Individual record code, such as 000368, are in accordance with the waveform codes given in the European Strong-Motion Database *ESD* [69].

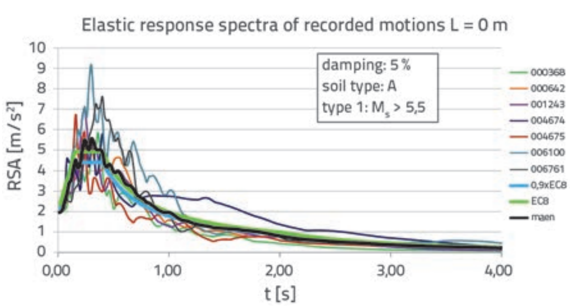

Figure 8. Elastic response spectra of seismic records needed for conditional simulation

As mentioned in Section 2, power spectral density functions and coherency functions are used in the procedure of conditional simulation for several stationary non-overlapping time windows. That is why stationary time windows of individual records, and the power spectral density function for every window, must be specified after definition of the coherency function. The time step of all acceleration time-histories is Δt = 0,01 s, and the number of time steps within individual stationary time window must be 2^n because inverse fast Fourier transforms are used in the simulation.

In every stationary time window T the acceleration time-history is defined as $a_j(t_l)$ where $t_l = \ell{\cdot}\Delta t$ ($\ell = 0,\ldots,N_T$) is the discrete time instant. An equal number of samples was used for the definition of the frequency and time step ($\ell = n = N_T$).

One set of recorded (L = 0 m) and generated (L = 100 m) ground acceleration time-histories, with the corresponding displacements, is shown in Figure 9.

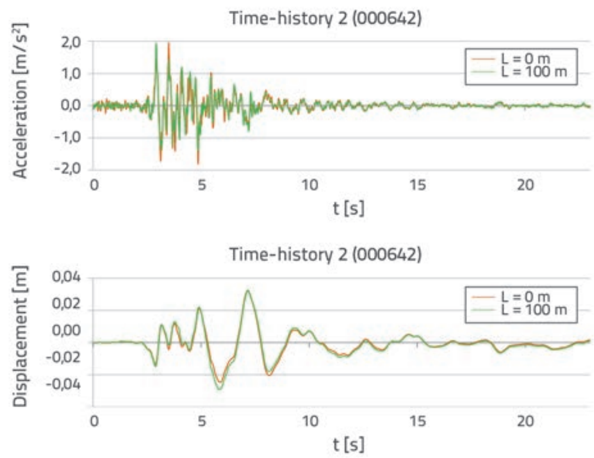

Figure 9. Recorded and generated ground acceleration time-histories with corresponding displacements obtained by conditional simulation procedure

The software program Seismospect [80] was used for the time-histories processing, and data processing was conducted according to guidelines given by Liao and Zerva [81]. Baseline correction was applied to all generated time-histories using the linear function. Frequencies were filtered using a Butterworth high-pass filter of fourth order, where the corner frequency was determined according to [81] and it amounts to f_c = 0,13 Hz for the generated time-histories of 20 s duration.

The Hilber-Hughes-Taylor direct integration method with α = 0 (abbreviated to HHTα) and with the time step of 0,001 s, was used for numerical execution of the analysis. In order to obtain the results of the highest possible accuracy in numerical analysis using displacement time-histories, it is advisable to set a relatively small time step. In fact, it is known that displacements are cubic functions, while accelerations are linear functions within each time interval, and so a smaller time step or a higher order solution must be used in time-history analysis using displacement records [60]. The Rayleigh damping is specified [54], where the damping of the first vibration mode, and the vibration mode for which the total mass participating ratio exceeds 90 %, amounts to ζ = 5 %.

The response to seismic action defined by 20 sets of time-histories (seven sets are obtained by conditional simulation and thirteen by unconditional simulation) was analysed for four types of excitation, which amounted to the total of 80 load cases. Only key results for the arch, which is regarded as the main load-carrying element of the bridge, are presented below.

3.3. Processing and analysis of results

The ratio of absolute peak response values for the spatially variable and uniform excitations was determined for each analysed arch section (springing, one-fourth of the arch span and crown) and for each set of time-histories (twenty in total) according to expression:

$$\rho = \frac{\text{peak response to the spatially variable excitation (COH,WP or COH+WP)}}{\text{peak response to the uniform excitation (UNIFORM)}} \quad (17)$$

If the ratio $\rho > 1$, then the spatially variable excitation exerts an unfavourable influence on the seismic response as compared to the uniform excitation. The values of axial forces N, shear forces T, bending moments M, and absolute horizontal u_x and vertical u_z displacements were analysed. Once the ratio ρ of all required values for each set of time-histories (20 sets x 3 types of spatially variable excitations) was determined, the arithmetic mean of the ratios was determined separately for each type of spatially variable excitations (COH, WP and COH+WP), as well as the standard deviation which is in all cases smaller than one third of the corresponding ratio ($\sigma < 1/3\ \rho$).

Figure 10 shows arithmetic means of the ratios separately for each type of the spatially variable excitation (COH, WP i COH+WP). The most unfavourable influence of the spatially variable excitation occurs for the bending moment at the crown due to the excitation COH+WP and amounts to as much as r = 3.63. The maximum displacement increase was registered for the vertical arch crown displacement due to COH excitation and it amounts to r = 1.59. A very unfavourable response in crown is expected, because the arch crown of a symmetrical bridge is not moving vertically under simultaneous translational seismic excitation of supports, as symmetric vibration modes are not excited, while both symmetric and antisymmetric modes are excited by the spatially variable excitation (Figure 11).

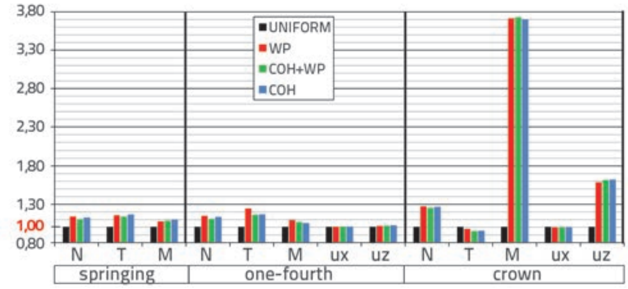

Figure 10. Arithmetic means of the internal forces and arch displacement ratios, for uniform and different types of spatially variable excitations

A uniform excitation provides results on the safe side for the shear force at the crown, and acceptable results (differences with respect to spatially variable excitation of less than ±10 %) for horizontal arch displacements, vertical displacement in one fourth of the arch span, and bending moments at arch springing and in one fourth of the arch span. The increase of shear force in one fourth of the arch span amounts to 1.23 for the excitation WP, and at the springing it amounts to 1.15 for the excitation COH. The greatest increase in axial forces with respect to the uniform excitation of supports occurs at arch crown for the excitation WP and amounts to 1.25, while it is 1.14 at the one fourth of the arch, and 1.13 in the springing.

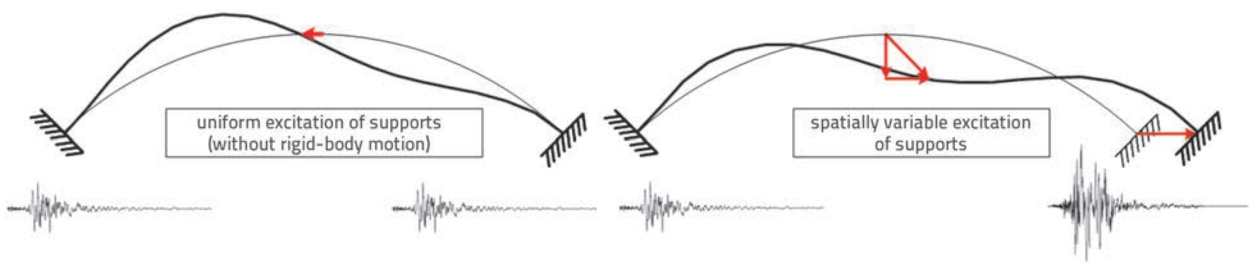

Figure 11. Relative arch displacements at time instant t = 5 s due to the uniform and spatially variable excitation of supports

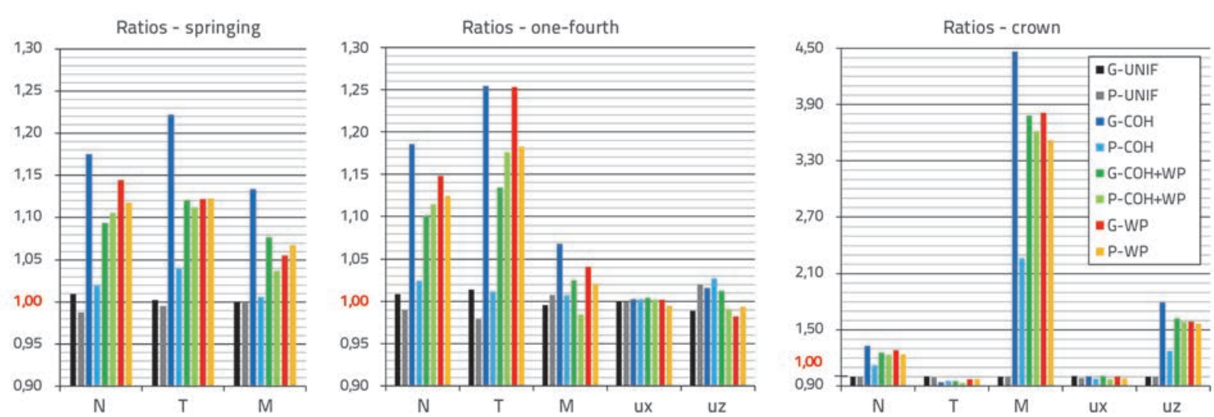

Figure 12. Arithmetic means of the internal forces and arch displacement ratios for spatially variable excitations dependent on the ground motion generation method

Arithmetic means of the ratios are presented in Figure 12 separately for the time-histories generated by unconditional simulation (marked G) and time-histories generated by conditional simulation (marked P).

Figure 12 shows that the use of time-histories generated using the unconditional simulation procedure mainly provides results that are on the safe side. However, it should be noted that these "artificial" motions are used due to lack of recorded ones and that they cannot realistically describe seismic excitation. The advantage of utilizing such simulations lies in a simple and rapid generation of a great number of samples (once the computer algorithm is made), which is useful in parametric analyses. The conditional simulation procedure based on the use of recorded time-histories is more demanding as it is, first of all, necessary to find a set of records in a strong-motion database that is appropriate for the location under study (which is not always possible), taking into account rules of valid standards and additional data processing (base-line correction and filtering). Then time windows and power spectral density function must be defined for each selected record considering that the number of time steps within an individual stationary time window equals 2^n because inverse fast Fourier transforms are used in the simulation. It is very difficult to fully automatize such procedure. Once the mentioned preliminary steps have been taken, the time-histories generation procedure can be conducted for other bridge supports. In this way, the generated samples retain original record properties (e.g., frequency range of the signal) and so they can simulate the seismic excitation more realistically when compared to "artificial" motions generated using a probabilistic model. Considering these findings, it is proposed that the ground motions generation procedure be selected depending on the complexity of the problem (e.g. design or theoretical parametric analyses). It should be noted that a special attention must be paid to the processing of acceleration time-histories, because displacement time-histories are usually used in software packages to define spatially variable excitations.

Results show that the influence of spatial variability of ground motion should not lightly be discarded and it is also significant to point out that the rules from valid standards for seismic design of bridges [67], which state that in case of approximately uniform soil properties along the bridge the spatial variability has to be considered when the bridge exceeds 400 m in length for rocky terrain, cannot safely be applied in the analysis of all bridges. The standard provides a similar approach to all load-carrying systems of bridges, although it is known that system properties greatly influence response of structures to spatially variable seismic excitations.

And, finally, the limitations of the described example should also be stated. As already mentioned, the two-dimensional analysis of the bridge was performed with the excitation set in the direction of bridge axis. In case of three-dimensional analysis of a structure using time stepping methods, the seismic excitation should be specified by the simultaneously acting ground displacement time-histories (at different supports) in three mutually orthogonal directions (longitudinal direction of the bridge, transverse direction of the bridge as related to the main axis, and vertical direction). During the time-histories generation procedure, it may be assumed that the motions in different directions are statistically independent, and therefore, simulations could be performed separately for each direction.

In addition, the effect of local-site conditions was not considered in the numerical example, because, due to great thrust, deck arch bridges are mostly built on rock. If the supports of a bridge were situated in different soil types, earthquake motions could be generated according to the scheme given in Figure 4, but these supports should be associated with the corresponding response spectra taking into account the soil category (Section 2.4). Although the considered coherency function in such simulations should account for changes in soil properties at supports, unfortunately, it does not exist as all coherency functions have been derived for homogeneous soil conditions only. Also, in specifying the wave-passage effect, it is assumed that seismic waves propagate in one direction at a constant apparent velocity along the ground surface, which is obviously not realistic for cases with an irregular underground topography. For a more complex approach to the modelling of local-site conditions, the reader is once again referred to [8, 10-12]. Taking into account the effort-intensive nature of such calculations and a low reliability of input data (seismic excitation, soil properties, etc.), for engineering practice it is sufficient to apply the presented standard approach based on the use of different response spectra for different soil types at the supports.

4. Conclusion

Spatial variability of ground motion accounts for differences in seismic motion at different locations on the ground surface. Earliest, simple studies of this phenomenon, at that time defined as delayed arrival of seismic waves to more distant supports of the bridge, were initiated in the 1960's. After arrays of closely spaced measurement stations were set up in different parts of the world to measure ground shaking during earthquakes, which resulted in highly valuable data, some other phenomena that induce spatial variability of ground motion were also observed. In addition, these data constituted a foundation for generation of the currently used spatial variability models.

With the advancement of technology, computer power and knowledge about the very phenomenon of spatial variability of earthquakes, the investigations have become more extensive and comprehensive. This paper is complemented with an extensive list of references presenting research about the response of bridges with various load-carrying systems to spatially variable excitations. Although many bridge response studies have so far been conducted, at the present state of knowledge it is still impossible to predict for individual types of structures whether the spatially variable ground motion will have a detrimental effect on the response, although this would be of special significance for the design. This is why respectable international organisations propose further research on the response of bridges of various layouts, oriented toward the engineering practice. It is interesting to mention that all new and existing larger bridges in California (in the scope of seismic retrofit program) have been analysed for spatially variable ground motions [82].

Although the European standard for seismic design of bridges proposes a simplified seismic analysis procedure for the spatially variable ground motion, the research made so far has pointed to relatively low applicability of the proposed procedure. That is why a special attention is paid in the paper to procedures for the generation of spatially variable time-histories that are used in the implementation of time stepping methods which, although time consuming and numerically demanding, constitute the most powerful tool for the seismic design of structures. In addition, to better explain this complex procedure, the methodology for the design and implementation is presented on the example of a reinforced concrete arch bridge. The time stepping method known as HHTα was applied, using seismic motions generated by conditional simulation methods with the original recorded time-history, and unconditional simulation method based on the probabilistic model, in accordance with the prescribed elastic response spectrum. Bridge response values due to uniform seismic excitation of supports were compared to the response values due to spatially variable excitation that included wave-passage and coherency loss effects. It was demonstrated on the studied 100 m span bridge that the phenomenon of spatial variability of ground motion has an unfavourable effect on the response of most of the considered design values in the bridge arch. At that, a special emphasis should be placed on the unfavourable action of spatially variable excitation at the arch crown where the increase in bending moment is as much as 3.63 times compared to the uniform excitation of supports. That is why spatial variability of ground motion (with all necessary effects) should be considered in the seismic analysis of similar load-carrying systems.

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