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Investigation of the effectiveness of nature-inspired buttress forms in supporting masonry structures

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Research paper - Subject review

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Investigation of the effectiveness of nature-inspired buttress forms in supporting masonry structures

A buttress is a crucial element for vaulted and domed masonry structures. In this study, after reviewing the buttress forms in historical masonry architecture, previous studies on masonry buttresses are summarized. Then, some ideas for potential studies on masonry buttresses are presented. Finally, the effectiveness of an idealized tree buttress-shaped buttress in supporting masonry structures against seismic forces is investigated. The results indicate that a concave parabolic buttress provides structures with a higher resistance than a buttress with an idealized shape of tree buttresses.

Key words:
buttress forms, historical masonry structures, inspired by nature, seismic resistance, vault

Pregledni rad

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Istraživanje učinkovitosti oblika upornjaka inspiriranog prirodom u podupiranju zidanih konstrukcija

Upornjak je važan element za nadsvođene i kupolaste zidane konstrukcije. Na početku ovog istraživanja, nakon pregleda oblika upornjaka u povijesnoj zidanoj arhitekturi, sažeta su dosadašnja istraživanja o zidanim upornjacima. Zatim su predstavljene neke ideje za buduća istraživanja zidanih upornjaka. Konačno, istražuje se učinkovitost idealiziranog upornjaka u obliku potpornog korijenja za podupiranje zidane konstrukcije protiv seizmičkih sila. Uočeno je da konkavni parabolični upornjak pruža veću otpornost konstrukciji nego upornjak s idealiziranim oblikom potpornog korijenja.

Ključne riječi:
oblici upornjaka, povijesne zidane konstrukcije, inspirirano prirodom, seizmička otpornost, svod

1. Introduction

Stone, brick, and adobe, which are called masonry building materials, have a privileged place in human history. Their easy availability and cost-effectiveness have enabled the construction of almost all types of structures for thousands of years. The walls of vaulted, domed, or arched structures experience significant thrust forces. In order for the walls of these structures to safely meet the horizontal component of the thrust they experience, they must be either fairly thick or supported by buttresses. Thick walls make buildings bulky and increase the amount of materials used to construct them. Hence, the use of buttresses is a more logical alternative. Buttresses are also extremely important and necessary for structures because they enable structures to withstand horizontal effects due to earthquakes and winds. An examination of historical masonry structures, especially those with high walls, reveals that most of them are supported by buttresses. This undoubtedly demonstrates that the architects and builders of these structures had a high awareness of the structural importance of buttresses.

Buttresses are generally defined as "projection built up against a wall or a structure, to form additional strength or furnish support" [1]. There is no definite information regarding when buttresses were first used in history. For most historical masonry structures, however, buttresses and the walls of the structure were constructed simultaneously; that is, they are original components of the structures. However, for some structures, at least some of existing buttresses were added later. Buttresses in building-type historical masonry structures can be divided into two main groups: those that are continuously interconnected with the wall (i.e. classical buttresses) and those that make contact with the wall over a small area and form an arch (i.e. "flying buttresses"). These buttresses transmit the thrust force from the wall to a strong pier-like buttress at the other end of the structure.

The world's great historic masonry architecture contains a variety of buttresses. This diversity will be first revealed in this paper with examples. Then, studies on masonry buttresses to date will be reviewed. As there are more studies to be carried out on masonry buttresses, recommendations for additional investigations will be presented. To the best of the authors" knowledge, this study explores, for the first time, the effectiveness of nature-inspired buttress forms in supporting masonry structures against seismic horizontal forces.

2. Buttress forms in building-type historical masonry structures

Most of the walls of building-type historical masonry structures are supported by buttresses. In this section, classical wall buttress forms and flying buttresses are discussed. Classical wall buttresses are classified as the most common, combined, less common, particular geometric shape, and special forms. Flying buttresses are studied collectively.

2.1. Classical wall buttresses

Statically, classical wall buttresses support the wall with their weight, therefore balancing the overturning moment created by the thrust of a vault, dome, or arch. These buttresses are continuously interconnected with the wall they support along their height.

2.1.1. Most common forms

Among buttresses, rectangular buttresses are the most common in historical buildings worldwide. Owing to their ease of construction and simple form, they have been used in almost all types of buildings. Figure 1 shows two buildings having rectangular buttresses.

Figure 1. Two historical buildings having rectangular buttresses: a) Kızıltepe Grand Mosque (Mardin, Turkey; Photo: Authors); b) Şarapsa Han (Inn) (Antalya, Turkey; Photo: Authors)

Figure 2. Examples of trapezoidal and triangular buttresses: a) Süleymaniye Mosque (İstanbul, Turkey; Photo: Authors; b) Piyala Paşa Mosque (İstanbul, Turkey) [3]; c) Sovana Cathedral (Toscana, Italy) [4]; d) Eziran Friday Mosque (Isfahan, Iran) [5]

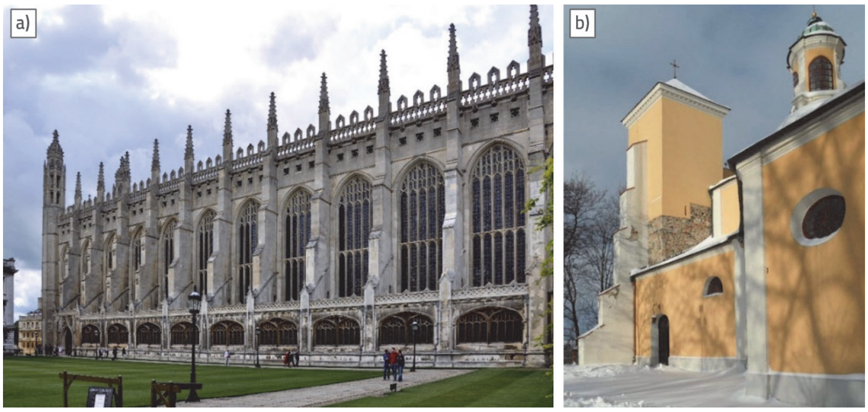

Figure 3. Examples of stepped buttress: a) King"s College Chapel (Cambridge, England) [6]; b) Basilica of the Assumption of Our Lady (Trzemeszno, Poland) [7]

The trapezoidal buttress has been one of the most preferred buttresses in historical buildings owing to its continuously increasing cross-section from top to bottom and therefore its contribution to preventing walls from overturning. For this type of buttresses, the cross-section variation is only along its depth (dimension perpendicular to the wall); however, there are also cases where variation exists both along the depth and in the thickness (dimension parallel to the wall). Figures 2a and 2b show two examples of buildings with trapezoidal buttresses. Although not as popular as rectangular and trapezoidal buttresses, triangular buttresses have also been used to support historic masonry buildings. Figures 2c and 2d provide two examples of triangular buttresses.

Another frequently encountered buttress type in masonry structures is the stepped buttress, which is commonly found in historical cathedrals of Europe. This type of buttress is rather prevalent, and two examples are provided in Figure 3. The popularity of this buttress type can be attributed to tradition and the low cost of the materials used to construct it. Moreover, its stepped shape enhances the facades of buildings.

2.1.2. Combined forms

Numerous historical masonry buildings possess combined buttresses consisting of two or more parts with different shapes. The combinations are diverse; some buttresses comprise of rectangular and triangular part, rectangular and parabolic part, or trapezoidal and rectangular part. Figure 4 presents two examples.

2.1.3. Less common forms

It has been observed that a few historical masonry structures have pure curvilinear (parabolic or other) buttresses. This might be because these buttresses have more frequent maintenance requirements, especially the concave ones, owing to problems arising from precipitation and frost. Existing curvilinear buttresses are either in concave or convex shape. Two examples are given in Figure 5. A review of historical masonry architecture reveals that curvilinear concave buttresses were mostly preferred in the upper parts of structures. These buttresses can be observed, for example, on the Cuetzalan del Progreso Church in Mexico (Figure 5a), Cadiz Cathedral in Spain, and Siena Cathedral in Italy. It can be predicted that, in addition to their aesthetics, these curvilinear concave buttresses were used in the upper parts of these structure because they do not obstruct the view owing to their form.

Figure 5. Examples of curvilinear buttresses: a) Cuetzalan del Progreso Church (Progreso, Mexico) [10]; b) Templo de Santo Domingo (Bolivia) [11]

2.1.4. Geometric shape forms

In the historical architectural buildings of the world, there are also buttresses with certain geometric shapes. Semi-cylindrical, lobed, triangular prism, half-conic, triangular pyramid, hollow cylindrical, and hollow triangular prism-shaped buttresses, as well as buttresses that resemble a portion of a polygonal prism, are examples of these types of buttresses. In Figure 6, some examples are presented.

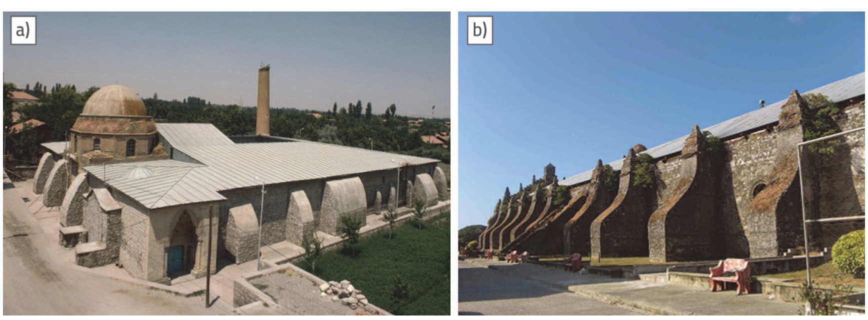

Figure 4. Two historical buildings that have combined buttresses: a) Malatya Grand Mosque (Malatya, Turkey) [8]; b) Paoay Church (Paoay, Philippines) [9]

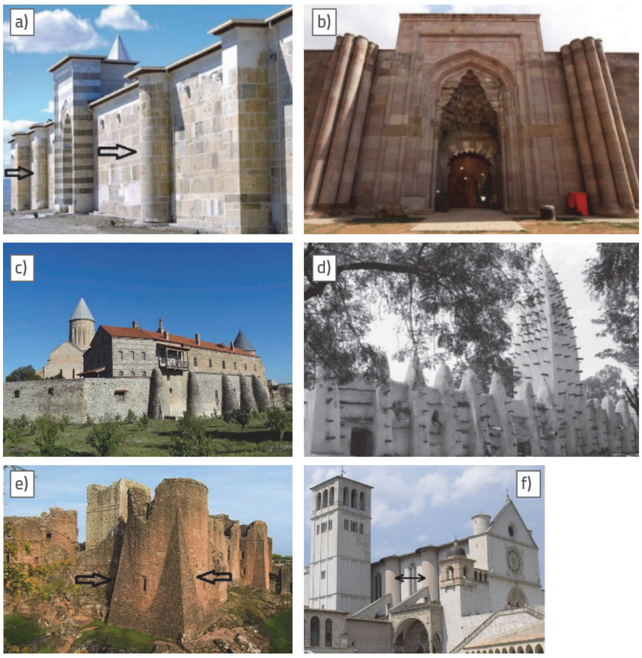

Figure 6. Special geometric shape buttresses: a) Zazadin Han (Konya, Turkey) [12]; b) Tuzhisar Sultan Han (Kayseri, Turkey) [13]; c) Alaverdi Monastery (Kakheti, Georgia) [14]; d) Great Mosque of Bobo-Dioulasso (Bobo Dioulasso, Burkina Faso) [15]; e) Goodrich Castle (Herefordshire, England) [16]; f) Basilica of San Francesco Assisi (Assisi, Italy) [17]

2.1.5. Special forms

Some buttresses in the world's architecture are considered special, but their numbers cannot be known. Although they appear to be similar, each one of them is actually unique. Figure 7 shows only four examples. Some of these buttresses, such as the snail-like buttresses of the dome of the Basilica of Santa Maria della Salute (Figure 7b), are aesthetically pleasing that it is impossible not to admire them and appreciate the people who designed and built them.

2.2. Flying buttresses

Flying or arch buttresses are commonly found on Gothic cathedrals. This buttress type, owing to its attributes, made the construction of magnificently high Gothic cathedrals possible. Some modified types of this buttress are also found in Islamic and other architectural styles. Figure 8 shows four examples of flying buttresses.

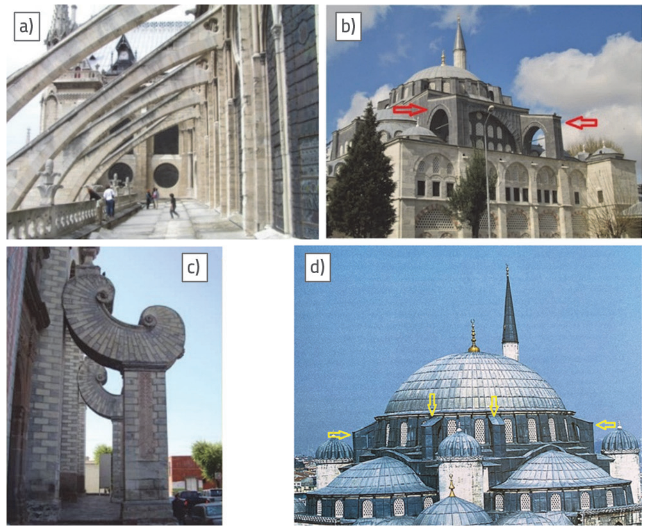

Figure 8. Examples of flying buttresses: a) Cathedral of Notre-Dame de Paris (France) [22]; b) Kılıç Ali Paşa Mosque (İstanbul, Turkey) [23]; c) Unique flying buttresses of Santa Rosa de Viterbo Church (Querétaro, Mexico) [24]; d) Dome flying buttresses of Şehzade Mosque (İstanbul, Turkey) [25]

Figure 7. Uniquely shaped buttresses: a) Corderie Royale, (Rochefort, France) [18]; b) Dome of the Basilica of Santa Maria della Salute and its snail-like buttresses (Venice, Italy) [19]; c) Cartuja de Nuestra Señora de las Fuentes Monastery (Huesca, Spain) [20]; d) San Francisco de Assisi Mission Church (New Mexico, USA) [21]

3. Previous studies on masonry buttresses

There are several valuable studies on masonry buttresses in the literature. The main studies are reviewed in this section. Ungewitter and Mohrmann [26] wrote a seminal two-volume book on the construction and behaviour of Gothic cathedrals, in which they devoted a vast portion to both classical and flying buttresses. In his work on thrust line theory, Milankovitch [27] demonstrated the application of the theory to various masonry buttresses. Sanabria [28] made comments on the Spanish architect Rodrigo Gil de Hontãnón's formulas for safe sizing of buttresses. Clark and Mark [29] undertook a comprehensive examination of the first nave flying

buttresses of Notre Dame de Paris. Huerta [30] and in another study [31] provided in-depth information about the sizing rules of buttresses during the medieval and Renaissance eras. He posits that ancient builders had a rather high level of awareness about the importance of buttresses for the safety of entire structures. In her detailed research on Anatolian Seljuk Caravanserais, Yavuz [32] provided a large amount of information on many aspects of buttresses in these structures, such as their form, plan shape, location, and decoration.
Heyman [33], in his legendary book *Stone Skeleton*, firstly discussed the structural theory of masonry structures and then explained the behaviours of various structural elements and parts, including flying buttresses in historic masonry structures. In his other two studies [34, 35], Huerta focused on the rules of buttress sizing, among other principal structural elements, in medieval structures, especially in Gothic churches and cathedrals. Ochsendorf et al. [36] dealt with the collapse of masonry buttresses under concentrated oblique loads, while Nikolinakou et al. [37] discussed the structure and form of early Gothic flying buttresses. Soudipour [38] attempted to explore the architecture of Mesopotamian temples from the Ubaid to the Old Babylonian period. His thesis reveals that builders of the oldest cities were aware of the buttress as a structural element and used it in their structures. Ochsendorf and De Lorenzis [39], considering various factors, such as sliding, limited compressive strength, and leaning, investigated the collapse condition of rectangular masonry buttresses subjected to a concentrated oblique load. García and Meli [40] attempted to determine whether the 16th century Mexican convent church builders followed structural rules noted in building treatises from that period when sizing structural elements, especially walls and buttresses.
Huerta [2] performed a detailed investigation on the safety of masonry buttresses. He first outlined the development of buttress design since the 16th century and then summarized state-of-the-art approaches in an analysis of modern masonry buttressed and discussed estimations of buttress safety. In the study, the contribution of buttress forms to stability against overturning was also briefly examined. Four buttresses that have equal heights and volumes shown in Figure 9 were considered, and it was expressed that, if the moment of stability of the rectangular buttress (Figure 9a) is taken as 1, the stability moments of others are 1.71, 1.63, and 2.18, respectively.
Considering fractures and possible sliding, De Lorenzis et al. [41, 42] performed detailed collapse analyses of trapezoidal and stepped masonry buttresses. They obtained both analytical and numerical solutions. Moreover, they compared the relative efficiency of different buttress shapes for a given volume.
Makris and Alexakis [43] carried out a comprehensive study on the fracture of masonry rectangular buttresses and towers, when they subjected either to a concentrated inclined load at their top or to inertia loading due to an earthquake. They determined that, in their limit state, slender masonry buttresses and towers collapse by rotating about their base corner, whereas less slender ones may fail owing to shear.

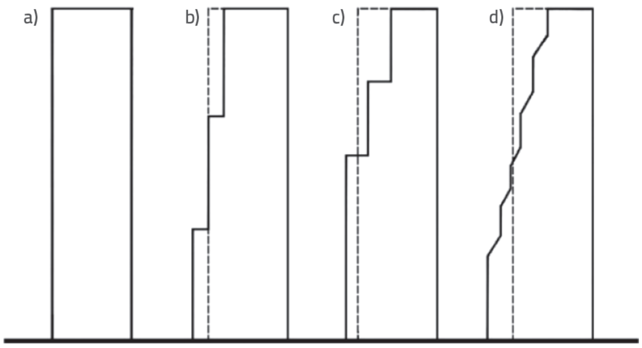

Figure 9. Buttresses that Huerta (2010) [2] considered when examining the influence of the buttress form on the moment of stability

Quintas [44], by following the concepts of classical Strength of Materials, analysed flying buttresses as rampant arches. In the study, after discussing issues such as passive thrust, yield patterns, and lines of thrust in rampant arches, the evolution of flying buttress in Gothic cathedrals was discussed in detail.
Karimi et al. [45], through library studies and descriptive and analytical research methods, investigated the typology and developments of buttresses in Iranian architecture. One of the main results of the study was that, over time, architects better understood the structural importance of buttresses and used this element more consciously in their structures.
Alexakis and Makris [46], by employing the principle of stationary potential energy, attempted to identify the lower hinging mechanism of a single-nave barrel vault supported on rectangular buttresses when subjected to lateral inertial loading in addition to gravity loading. Kavanaugh et al. [47], using distinct element modelling, performed a case study on pier buttresses of the Amiens Cathedral to validate the usability of graphical thrust line analysis. In their study on earthquake-resistant geometrical properties of churches, public buildings, and colonial houses in Santiago (Chile), Jorquera et al. [48] emphasized the role of buttresses as well as other features in the earthquake resistance of these structures. Como [49] considered classical and flying buttresses as well in his book on the statics of historical masonry structures. With regard to classical buttresses, he examined inclined detachment cracks during overturning and buttress side strength. Flying buttresses were also discussed.
In her study on the mechanics of flying buttresses in the case of the Mallorca Cathedral, Fuentes [50] first provided an overview of the structural behaviour of flying buttresses within the framework of limit analysis. She then analysed the deformations and cracks of flying buttresses in the Mallorca Cathedral and studied problems such as sliding of flyers" head and different solutions adopted for them throughout history. Vannucci et al. [51] examined the wind strength of Gothic cathedrals such as the Notre Dame de Paris. In the study, they considered the global collapse of the cathedral. In addition, they determined that partial but significant local damage could occur

to different parts of the cathedral, including its flying buttresses. İzol et al. [52], utilising a numerical calculation procedure and adopting an equivalent static analysis method, investigated the out-of-plane seismic resistance of high masonry walls having rectangular buttresses. Marrs [53] aimed to determine the out-of-plane strength of stone walls having buttresses to contribute to knowledge available to civil engineers working on stone buildings. She accomplished this by surveying a series of historical churches in Ottawa (Canada), analysing historical references, and conducting a literature review on relevant modern research. One of the main conclusions of her study was that the historic method of approximating the strength of walls and buttresses by the overturning of a rigid body significantly overestimates the strength of these elements, especially if they are slender. In addition, she found that there is a significant increase in strength if the buttress is fully connected to the wall. In addition to the aforementioned studies, a number of other articles and conference papers on masonry structures have focused on buttresses. Elyamani et al. [54], for example, investigated the buttresses of the Mallorca Cathedral, which they took as an example in their study on seismic safety assessment of historical structures using updated numerical models. Furthermore, in their study on traditional earthquake-resistant techniques for vernacular architecture, Ortega et al. [55] devoted a section to the role buttresses play in counteracting horizontal loads. Some of the studies cited above are about classical buttresses, some are about flying buttresses, and some are case studies. Moreover, some theses and books related to the subject have been published. Although all these studies are truly valuable, as explained in the next section, there are many more issues regarding masonry buttresses that can be explored.

4. Other studies that can be carried out in the future on masonry buttresses

This section presents some novel ideas for future investigations on masonry buttresses. They are mostly related to classical buttresses.

1) In previous studies, buttress forms, analysed under a concentrated inclined load, representing the load from a vault (or arch), were restricted only to rectangular, trapezoidal, and stepped forms (Ochsendorf et al. [36], Ochsendorf and De Lorenzis [39] (2008), De Lorenzis et al. [41-42]. Although these buttresses are the most common, as discussed in Section 2, there are many other buttress forms that exist in historical masonry architecture. Although it is not possible to consider all of these, there are several types that should be studied. Some of them are shown in Figure 10. A simple masonry wall-buttress model having these buttresses individually can be analysed under an increasing concentrated inclined load. In this manner, the effectiveness of each buttress form can be determined. In order for these calculations to be meaningful, the volumes of the buttresses must be kept constant in the analysis. In addition, the effectiveness of these buttress forms in offering horizontal resistance to the model can be investigated by performing pushover analyses, for example, in a perpendicular direction to the walls of a simple vaulted model.

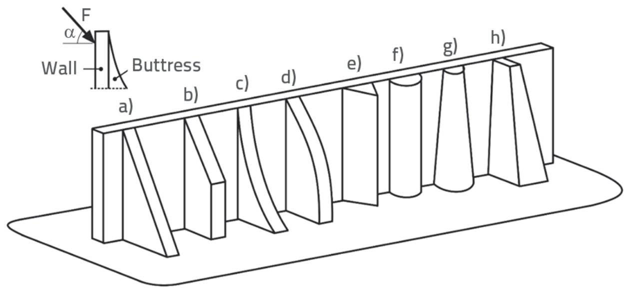

Figure 10. Some other buttress forms that should be considered in addition to rectangular, trapezoidal and stepped buttresses for the analysis under a concentrated inclined load: a) triangular buttress, b) a combined buttress, c) curvilinear concave buttress, d) curvilinear convex buttress, e) triangular prism buttress, f) semi-cylindrical buttress, g) truncated half-conic buttress, and h) trapezoidal buttress with varying thickness

2) The above two groups of investigations can also be performed while keeping the base depths of the buttresses constant.
3) In almost every country, there are many historical masonry structures having high walls supported by buttresses and covered with vaults or domes. Structure-specific studies can be carried out on these structures. The contribution of the buttresses can be determined clearly by performing static calculations and through comparisons of the current state (buttressed state) and un-buttressed state of the structure. Similarly, the effects of these elements on the dynamic behaviour of the structure can be determined by carrying out time-history analyses using the records of earthquakes that are important for the region where the structure is located. For example, the authors plan to do the above-mentioned studies on the Melik Mahmut Mosque (Mardin, Turkey), which is shown in Figure 11a, in the near future. On the facade of the building, there are three rectangular buttresses, the upper parts of which are decorated with hemisphere-like structures. The middle buttress is smaller than the side buttresses in terms of both cross-section dimensions and height. This feature of the building is noteworthy, emphasizing the importance of such a study.
As it can be understood, there is a great potential in the world for studies that can be performed such structure-specific.
4) In some historical masonry buildings, classical buttresses have the same height as the walls, but others are shorter than the walls. The effect of the height of the buttress relative to the wall on the behaviour of a building can therefore be investigated. The investigation can be realized, for example, using a simple masonry building model. Different buttress

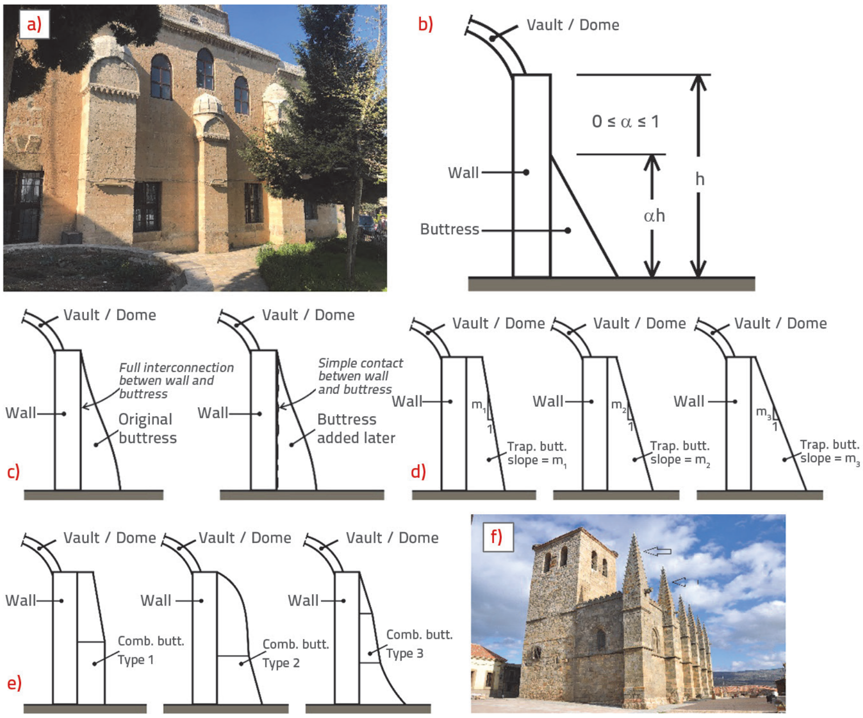

Figure 11. Various studies that can be carried out on masonry buttresses and masonry structures having buttresses: a) Example of a special historical masonry building having buttresses (Melik Mahmut Mosque, Mardin, Turkey), (Photo: Authors); b) Investigation of the effect of the height of a buttress relative to that of a wall on the behaviour of the structure; c) Investigation of the effect of age of a buttress on the behaviour of a structure; d) Investigation of the effect of the slope of the trapezoidal buttress on the behaviour of a structure; e) Investigation of the effectiveness of different combined buttresses; f) Historical masonry building example having pinnacles on its buttresses: Church of San Martin de Bonilla de la Sierra (Avila, Spain) [56]

forms can be considered. For a selected buttress form such as rectangle or triangle, while keeping the depth and thickness constant, the height can be changed, and analyses can be made. One side of a schematic model having triangular buttresses is shown Figure 11b. Thus, the effect of the buttress height on the results can be determined. Certainly, examinations can be carried out under both static and dynamic conditions.

5) For most historical masonry buildings, the buttresses are as old as the buildings; however, for some buildings, at least some of the buttresses were added later. Therefore, the effect of perfect (original buttress case) and weak (later added buttress case) connection states between a wall of a structure and buttress on the behaviour of the structure can be investigated. The investigation can be made on a simple masonry building model of a buttress form, and its dimensions can be selected. One side of a schematic model is shown in Figure 11c. In the case of buttresses that are added later, it can be considered that there is only a simple contact between the wall and the buttress. A static analysis (i.e. an analysis conducted while considering only the weight of the model alone, such as a non-linear static analysis in the direction perpendicular to the plane of the wall) can be performed. Thus, the effect of the connection state between a wall and buttress on the behaviour of the structure can be determined. In addition, calculations and comparisons to be made for the plain and buttress-added state of the model will reveal the level of support provided to the structure by adding buttresses.

6) As expressed, the trapezoidal buttress is one of the most widely used buttresses in historical masonry structures. For a constant volume value, numerous trapezoidal buttresses of the same thickness but with different slopes can be obtained. Therefore, the effect of buttress slope on building behaviour can be determined using a simple masonry building model by considering trapezoidal buttresses with equal volumes and thicknesses but different slopes (Figure 11d). Rectangular and triangular buttresses, which are the limit states of the trapezoidal buttress, can then also be studied.

7) It has already been stated that combined buttresses are among the most frequently encountered buttresses in historical masonry structures. By devising various forms of combined buttresses having equal volume and thickness, scholars can study the effect of buttress combinations on the structural behaviour using a simple masonry structure model. Some types for combined buttresses are given in Figure 11e.

8) Some historical masonry structures, such as the Church of San Martin de Bonilla de la Sierra (Avila, Spain; Figure 11f), have pinnacles on the classical wall buttresses. It is indisputable that pinnacles increase the aesthetics of a building. As these towers put additional load on the buttresses, they will also have an effect on the structural behaviour of the building. This effect can be studied using a simple masonry building model having buttresses, considering the different weights of pinnacles on buttresses. Both static and dynamic analyses can be performed. The effects of pinnacles on the pier buttresses supporting the flying buttresses on the structural behaviour of the structure have already been investigated (see, for example, Kavanaugh et al. [47]).

9) Although few in number, tower-type buttresses can be found on some historical structures This type of buttresses should also be examined. The effects of a normal rectangular buttress and several tower-type buttresses of equal volume can be compared (Figure 12). Static calculations that will be performed for example on a vaulted masonry building model will reveal how these buttresses affect the static behaviour of the structure. If they performed, the pushover analyses will show the effectiveness of the buttresses on the horizontal resistance of the structure.

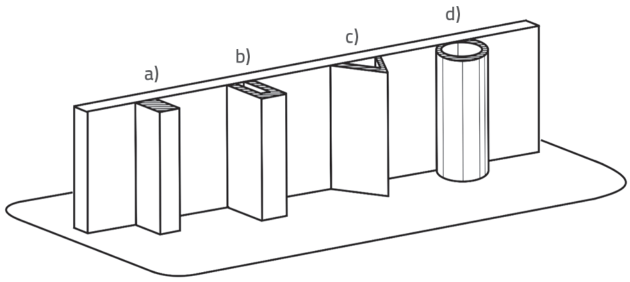

Figure 12. Rectangular buttress and various tower-type buttresses of equal volumes

10) The trunks of most trees form a sharp cornered notch with the ground surface. However, in some trees, this place, especially the windward side, is bridged by a pronounced root spur (buttress), and the sharp corner is defused. Such a tree is observed in Figure 13a. Inspired by the shape of buttress roots of such trees, Mattheck et al. [57] proposed the tensile triangles method (TTM), shown in Figure 13b, for easing the cracks in various technical components. Buttress roots of trees function, in general, such as tension ropes. Even so, in relation to our subject, their shapes seem worthy of consideration in determining the most appropriate buttress forms in various structures, such as masonry buildings, dams, and retaining walls.

Figure 13. a) Tree having buttress roots [58] and b) tensile triangles developed by Mattheck et al. [57] inspired by buttress roots for easing cracks in technical components

11) Studies on the experimental determination of the strength and behaviour of masonry buttresses and buttressed structures under static and dynamic loadings are lacking. Such studies would make valuable contributions to the subject. In addition, comparing the results of theoretical studies with experimental results will enable better interpretation of the findings obtained from them. Considering also the ideas listed above, it is needless to say that this experimental study field has a wide potential to work.

12) For flying buttresses, the effect of the slope of these buttresses on the behaviour of structure can be studied in detail. Moreover, by considering a flying buttress with a certain slope, the effect of applying a single load in the middle or a uniformly distributed load on the behaviour of the buttress is worth investigating. The large domes of most historical masonry structures are supported laterally at the hoop level by flying buttresses (Figure 8d). The level of support they provide to the dome can be determined by performing static analysis for the presence and absence of these buttresses and by comparing the results.

As will be appreciated, the ideas presented above are about issues worth working on.

5. Investigation of the effectiveness of buttresses in the form of tree buttress roots in supporting a masonry structure against lateral seismic forces

In this section, one of the ideas presented above is considered, and the results of a research study are discussed.
As mentioned in item 10 above, some trees have buttress roots. In windy conditions, the windward buttresses of such trees are exposed to tension, and those on the other side are subjected to compression (Figure 14). Buttress roots are effective in withstanding tension and function as tension ropes. There is of course a prevailing wind direction in the area where a tree is located; however, the wind sometimes blows from other directions. Furthermore, as the tree oscillates in breezes, the same buttress root is alternately subjected to tension and compression many times. Thus, a buttress root unavoidably undergoes both tension and compression.

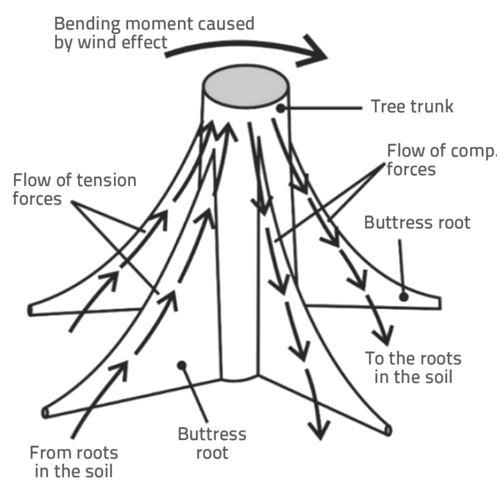

Figure 14. Depiction of the function of buttress roots (the drawing is based on Crook et al. [59])

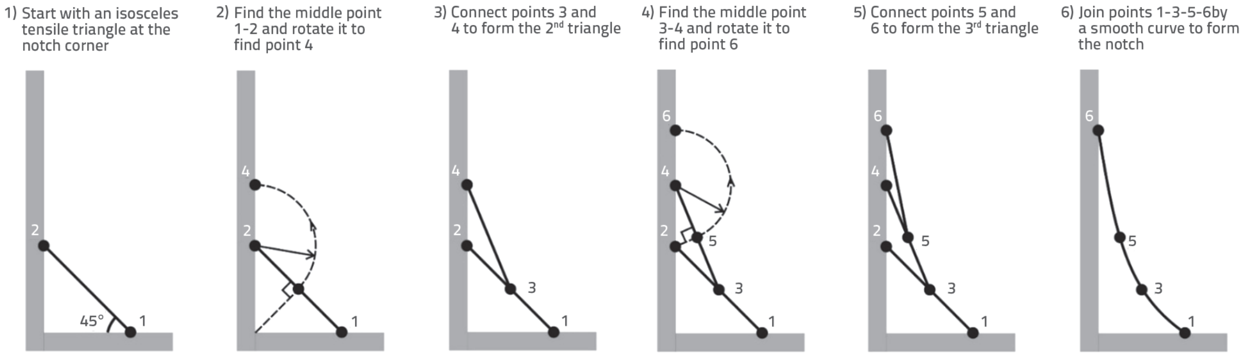

Figure 15. Mattheck et al.'s [57] tensile triangles method for forming a smoother notch using three triangles (the drawing is based on Abhyankar et al. [63])

Table 1. Dimensions of the considered buttresses (in m)

Buttress form	Height, h	Base depth*, d	Thickness**, t
Rectangular	11.10	2.20	1.35
Triangular	11.10	4.40	1.35
Parabolic	11.10	6.60	1.35
Buttress root shaped	2.35+2.84+5.91=11.10	4.70	1.983
* Dimension perpendicular to the wall, ** Dimension parallel to the wall.			

Table 2. Material properties used in the calculations

Unit weight, γ [kN/m³]	Modulus of elasticity, E [MPa]	Compressive strength, f_c [MPa]	Tensile strength, f_t [MPa]	Poisson's ratio, ν
23.37	11738	15.65	1	0.20

Buttress roots have held the interest of biologists and engineers for many years. Inspired by buttress roots, engineers have carried out shape optimization studies on various technical components, such as trusses and notches (Mattheck [60], Mattheck et al. [57], Mattheck et al. [61], Mattheck et al. [62], Abhyankar et al. [63], Yap et al. [64], Yap et al. [65], Feng et al. [66]). Here, the authors examine the effectiveness of only one type of tree buttress root form, namely the idealized tree buttress root form in the TTM proposed by Mattheck et al. [57], in supporting a masonry structure laterally. The TTM, which is a graphical method, was developed as a design tool that mimics the structures in nature (specifically trees) for the shape optimization of various technical components with respect to increased lifetime and reduced weight. The basis of the method is given in Figure 15.

For the analyses, a slightly modified model of Şarapsa Han (Inn) in Antalya, Turkey, which is a historic stone masonry building (Figure 1b) was used by the authors. The Han was preferred because of its simple and regular form. However, as its height-to-width ratio (aspect ratio), without including the buttresses, is approximately 0.55, it is considered a squat building. Such structures may collapse by shearing under the influence of increasing horizontal loads (Makris and Alexakis, [43]). Such a collapse case was beyond the scope of this study, and only the bending collapse case was investigated. Accordingly, the original model of the building was modified so that its height-to-width ratio was 1 without including the buttresses. However, all other features were left as they are. The model obtained using this approach is called "the basic model" hereinafter.

For the basic model, in addition to the tree buttress root-shaped buttress idealized by TTM, rectangular, triangular, and concave second-order parabola-shaped buttresses were also mounted separately for comparison, and calculation models were established. As reviewed in Section 2, a rectangular buttress is the most common buttress, and a triangular buttress is one of the widely encountered buttresses. Meanwhile, the curve of the concave parabolic buttress fits very well with the outer boundary of many tree buttress roots. Therefore, these three buttresses are considered in the comparison. The geometric properties of the buttresses whose volumes are maintained as 33 m³ are given in Table 1, and the material properties used in the calculations are presented in Table 2. The $\sigma - \varepsilon$ graphic of the material is given in Figure 16.

The material properties were obtained from an as-yet unpublished study performed by a research team including the first two authors of this study, which again considered Şarapsa Han as the sample structure. As expressed above, Sarapsa Han is a stone masonry building. In the mentioned study, the values given in Table 2 were obtained by considering the properties of the stone and mortar of the building and using the macro modelling (homogeneous material) approach.

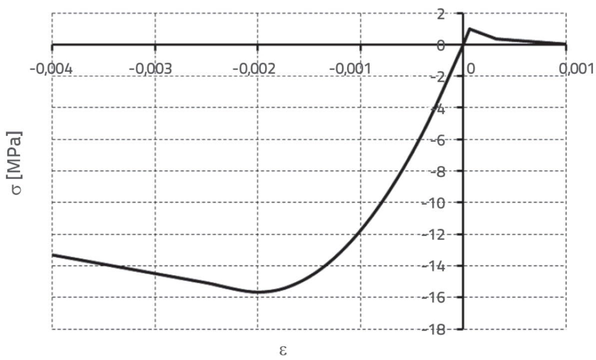

Figure 16. Uniaxial σ – ε graphic of the homogenized masonry material of the structure considered

Pushover analyses perpendicular to the plane of the longitudinal walls of the building (Figure 17) were performed on the models having the considered buttresses, and the effect of the buttress form on the transverse seismic resistance was investigated. In the calculations, Abaqus" [67] finite element program was employed. The C3D8R solid element was used to form the finite element meshes of the models. The meshes, numbers of elements, and nodes of one half of the basic model and half of each buttress are given in Figure 18. Note that the final meshes were formed by refining the meshes obtained at the beginning, until stable results were attained.

Figure 17. Building considered (Şarapsa Han, Antalya, Turkey) and direction of the pushover analyses carried out on the models of the building (Photo: Authors)

The most popular material models for quasi-brittle materials (e.g. concrete and masonry) are the smeared cracking model, brittle cracking model, and concrete damaged plasticity (CDP) model. The smeared cracking model is only suitable for monotonic loading and low compression conditions. Moreover, this model is mesh-sensitive with respect to the size and shape of the finite elements. Therefore, its applicability is limited. Meanwhile, the brittle cracking model can be used in any loading condition; however, it assumes linear elastic material behaviour in compression. For this reason, it is reliable only when the tensile failure dominates the brittle material behaviour. The CDP model is a more advanced quasi-brittle material model incorporated in Abaqus that considers crushing under compression and cracking in tension and can be used for any loading situation.

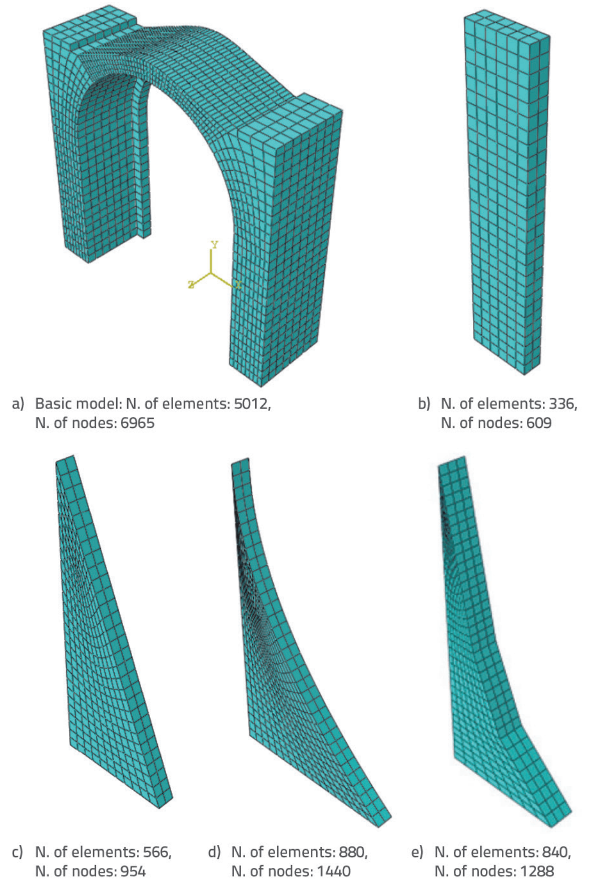

Figure 18. Finite element meshes and numbers of elements and nodes of half of the basic model and half of each considered buttress: a) rectangular buttress, b) triangular buttress, c) concave parabolic buttress, and d) Mattheck et al.'s [57] idealized tree buttress root-shaped buttress (figures are not to scale)

There are invaluable studies on the nonlinear analysis of masonry structures that used the CDP model successfully (see, for example, Tiberti et al. [68], and Valente and Milani [69]). Therefore, we also used the CDP model in this study. When using the CDP model, the necessary parameters to accurately simulate nonlinear behaviour are the angle of dilation ψ, the ratio of initial biaxial to initial uniaxial compressive strength f_{b0}/f_{c0}, the yield potential eccentricity ε, the ratio of the second stress constant on the tension meridian to the second stress constant on the pressure meridian K_c and the viscosity parameter μ. They are given in Table 3. These are frequently used values in the analysis of masonry structures similar to the one in this study with the Abaqus

Table 3. Damaged plasticity parameters used in the analyses

Dilation angle, ψ	f_{b0}/f_{c0}	Eccentricity, ε	K_c	Viscosity parameter, μ
10°	1.16	0.1	0.667	0.002

program (see, for example, Valente and Milani [69]). Two other damage parameters, d_t and d_c, expressing the degradation in the modulus of elasticity in tension and compression, respectively, were determined using the d = 1-σ/f equation, which is highly preferred in the literature. In this expression, f is the peak stress, and σ is the post-peak stress.

The pushover analyses were carried out in two steps. In the first step, the weights of the models were applied to the models, and in the second step, a uniformly applied mass proportional to the loading pattern was incrementally applied to the models up to the maximum lateral resisting capacity and until a failure mechanism emerged. For each model, a capacity curve (i.e. the curve of the base shear force versus lateral displacement of a preselected control node) was obtained. The middle node on the top of the vault was chosen as the control node.

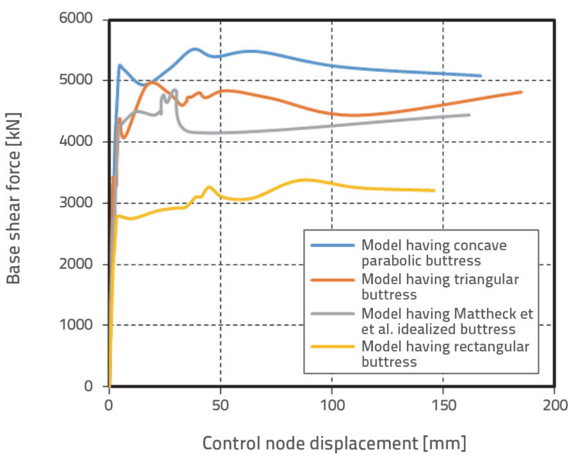

Figure 19. Capacity curves of the models having different buttress forms

The capacity curves obtained after the analyses for the models are presented in Figure 19, and the counter representations of the damage at the final stage of the analyses are given in Figure 20. As shown in Figure 20, the damage in the models is concentrated in four regions in general during the collapse phase. Meanwhile, as shown in Figure 19, the model having rectangular buttresses has the lowest seismic resistance, and the model having concave parabolic buttresses has the highest resistance. Although the triangular buttressed model has slightly higher resistance, the models having triangular and idealized buttress root-shaped buttresses yielded maximum seismic resistances that were close in value to each other. The results of the analysis described in this section revealed that the idealized buttress root-shaped buttress has no superiority over the triangular buttress, which is already widely encountered in historical masonry structures. However, the concave parabolic buttress, which represents the curvilinear geometry of the buttress roots of at least some trees well, is different from the other buttresses in terms of seismic resistance it provides to a structure.

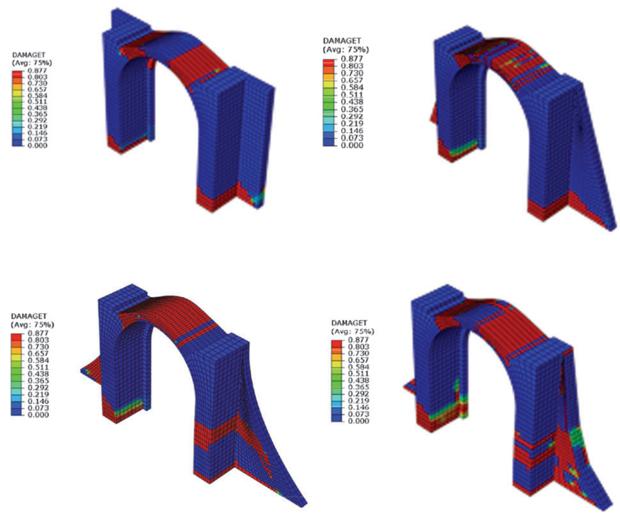

Figure 20. Contour representations of damage at the final stages of the analyses for different models,: a) model having rectangular buttresses, b) model having triangular buttresses, c) model having concave parabolic buttresses, and d) model having Mattheck et al.'s [57] idealized tree buttress root-shaped buttresses

Table 1 reveals that the concave parabolic buttress has the largest base depth among the four buttresses, and this is the primary factor responsible for this buttress" ability to provide the highest resistance to a structure. The roots of some trees with buttress roots have this structure. Therefore, trees with long and short buttress roots should be examined separately. Moreover, note that different buttress root shapes may be more suitable for different technical applications. The results of simple analyses have revealed that, for example, a concave parabolic buttress model inspired by long-base distance buttress roots rather than Mattheck et al.'s [57] idealized short-base distance tree buttress root model is more effective for engineering structures having buttresses, at least under certain loads and conditions.

Buttresses similar to concave parabolic buttresses, one of the buttresses we examined, were also used in historical masonry architecture. An example was given in Figure 5a, and another two examples are presented in Figure 21. Concave curvilinear buttresses are also used in modern architecture. Two examples are given in Figure 22. Buttresses having such a shape not only support structures but also give them an elegant appearance. Evidently, the development of forms for technical applications that are inspired by nature is definitely a subject that requires much more work. Moreover, these studies should be carried out separately for different structure types, such as reinforced concrete multi-storey buildings, buttress dams and concrete retaining walls with buttresses, as safety requirements, loads, and constraints differ depending on the type of structure.

Figure 21. Two historical masonry structures having concave curvilinear buttresses: a) Cathedral of Cadiz (Spain) [70]; b) Cathedral of Siena (Italy) [71]

Figure 22. Two modern buildings having concave curvilinear buttresses: a) Sea Monarch Condominiums, Pompano Beach, Florida [72]; b) Apartment building in Toronto, Canada (Building Blocks Magazine) [73]

6. Conclusion

Buttresses are crucial for the safety of historical masonry buildings, as evidenced by the prevalence of buttresses on historical buildings, some of which are very ancient. This study first revealed the diversity in buttresses used in historical masonry architecture. Results indicate that buttresses ranges from the simple but most common rectangular buttress to the snail-like buttress. Then, previous studies on both classical and flying buttresses were presented. It was revealed that rectangular, trapezoidal, and stepped buttresses are well studied statically under an inclined load acting close to their tops. It was also observed that there are many studies on flying buttresses. The study also offered many new ideas for future studies on masonry buttresses. In this regard, this study serves as a source of inspiration for researchers who will work on the subject in the future. Finally, the effectiveness of an idealized buttress root form, which was inspired by the buttress roots of trees, in supporting a masonry structure against seismic forces in the transverse direction was investigated. It was observed that the concave parabolic buttress, which better represents the buttress root forms of some trees, provides higher seismic resistance than the idealized buttress root-shaped buttress available in the literature. The study also revealed that we can draw inspiration from nature to develop buttress forms for various structures, as in many other subjects. It should not be forgotten that, in this regard, trees and countless other structures and shapes in nature, such as the shape of a mountain slope, the shape of a dam built by an otter, or the shape of an animal's hoof, can offer us inspiration.

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