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Dynamic identification of three 19th century churches in Turkey

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Professional paper

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Dynamic identification of three 19th century churches in Turkey

It is of great importance to determine the in-situ wall strength and dynamic characteristics of historic structures by experimental methods before structural analysis. The most effective experimental methods are the flat-jack, shear and vibration tests. After experimental methods, dynamic behaviour of the structures should be checked by the finite element analysis. Dynamic characteristics of three churches constructed in the 19th century in Balıkesir and Bursa provinces, now reduced to load-bearing walls (outer walls) only, are analysed in this study. It was established that, in order to identify dynamic characteristics of such structures, both the whole structure and the individual walls should be tested and evaluated by means of the finite element analysis.

Key words:

masonry structures, material tests, structural health monitoring, dynamic behaviour

Stručni rad

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Dinamička identifikacija triju crkava iz devetnaestog stoljeća u Turskoj

Prije samog proračuna konstrukcije povijesnih građevina važno je eksperimentalnim postupcima odrediti čvrstoću njihovih zidova isto kao i dinamičke karakteristike konstrukcije. Najdjelotvornije eksperimentalne metode su ispitivanje primjenom plosnatih preša, ispitivanje posmika te ispitivanje vibracija. Nakon eksperimentalnih metoda, treba se ispitati dinamičko ponašanje građevina pomoću analize metodom konačnih elemenata. U ovom se radu analiziraju dinamičke karakteristike triju crkava izgrađenih u 19. stoljeću u pokrajinama Balıkesir i Bursa od kojih su sada ostali samo nosivi (vanjski) zidovi. Utvrđeno je da se za definiranje dinamičkih karakteristika takvih građevina treba metodom konačnih elemenata ispitati i ocijeniti kako čitava građevina tako i pojedinačni zidovi.

Ključne riječi:

zidane građevine, ispitivanje materijala, praćenje stanja konstrukcija, dinamičko ponašanje

1. Introduction

Historic buildings, being our cultural legacy for hundreds of years, are extremely important and valuable in terms of cultural heritage. Transferring these structures from today to the future is an inevitable task for us. These structures have been affected over time by fires, earthquakes, wars, and many interventions. Thus, it is important to diagnose the current condition of these structures by in-situ tests and computational analyses so that they can be restored in an appropriate way. The decisions taken by the International Council of Monuments and Sites (ICOMOS) during an international workshop in 2003 constitutes the basis of this study. Before conducting restoration studies, the current condition, construction techniques, interventions, boundary conditions, structural properties and wall strength of the buildings, should be determined by in-situ tests. After this stage, a safety assessment should be performed before defining the intervention techniques [1].

It is of great importance to accurately determine the strength of masonry walls and dynamic characteristics of the real structure before the actual computer analysis. Dynamic parameters can be determined by modal analysis of the finite element models prepared based on boundary conditions and material properties. These parameters may be far from their expected values due to the loss of strength of the building materials over time, workmanship errors at the construction phase, cracks caused by various loads, and fatigue caused by collapse of the structure. Therefore, dynamic parameters of the structure should be determined by experimental and analytical methods [2].

The most effective methods for determining the wall strength are the flat-jack and shear tests. These test methods have been used by researchers for many years to accurately determine the wall strength of historic buildings [3-8]. An experimental method known as the Operational Modal Analysis (OMA) has often been used over the last two decades to determine dynamic behaviour of real structures. In this non-destructive test method, vibration tests are performed by placing sensitive accelerometers in perpendicular directions. Natural frequencies, mode shapes, and damping ratios of the structure can be determined by means of this test technique. However, this method has both advantages and disadvantages when applied to historical structures. Thus, it has the following advantages: it is a fast and cheap testing technique, there is no need to use an excitation equipment, the structure can be tested under operating conditions, and it can be used for damage detection of structures. On the other hand, it has the following disadvantages: modal participation factors cannot be computed, modal parameters can be difficult to determine in the presence of spurious harmonics near natural frequencies, use of very long cable transducers, and difficulties with the positioning of accelerometers. To ensure accuracy of dynamic parameters, ambient vibration

tests must be considered with modal updating techniques. Repeated tests are important for the evaluation of dynamic parameters. The orthogonality between the modes can be checked using the Modal Assurance Criteria (MAC) and the complexity plots [9-13]. Many studies have so far been made using the Operational Modal Analysis tests to determine dynamic parameters of structures [14-20].

Dereköy and Aydınpinar churches in Bursa and Ballıpinar Church in Balıkesir, all built in the 19th century, are examined in this paper by means of experimental and finite element analyses. The compressive and shear strength of the walls is calculated using the flat-jack and shear tests. Natural frequencies and mode shapes of the entire structure and individual walls in x direction are analysed by means of the Operational Modal Analysis tests. The data obtained from all experimental approaches are used in the finite element models. The experimental and finite element analysis results are compared.

2. Architectural features of the churches

As a result of abandon and neglect, the studied churches have not been in use for more than two decades now. The roof of the churches and all timber pillars and beams related to the roof structure collapsed, so that only the load bearing walls (outer walls) are still standing today. Architectural features of the churches were examined in terms of construction dates, material usage, dimensions of the churches, and height of the walls. Aydınpinar Church was built between 1846 and 1870. Brick, rubble stone, and cut stone materials were used in the construction of the walls. Lime mortar was used as binder. The structure occupies an area of 15,85 x 25,30m. Outside walls are 0.70 m in average thickness. The wall height is 7.82 m. Dereköy Church was built in 1857.

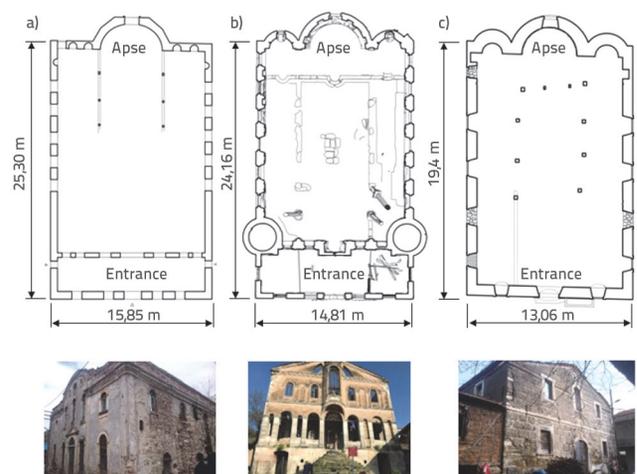


Figure 1. Plan views and photos of entrance facades of: a) Aydınpinar Church, b) Dereköy Church, and c) Ballıpinar Church

Brick, rubble stone and cut stone materials were used in the construction of the walls. The building occupies an area of 14,81

Table 1. Photographs of exterior, interior and wall section of churches

Churches	Exterior of the church	Interior of the church	Wall section
Aydınpinar Church			
Dereköy Church			
Ballıpınar Church			

x 24,16 m. Outside walls are 0.85 m in average thickness, and the wall height is 9.00 m. Ballıpınar Church was built in 1895. The structure occupies an area of 13,06 x 19,04 m. Stone and brick were used in the construction of the walls. The walls are 0.80 m in average thickness, and the wall height is 6.80 m. The plan views and photographs of the entrance facades of the churches are presented in Figure 1 [21, 22]. Photographs of the exterior, interior and wall section of the churches are presented in Table 1.

3. Experimental studies of the churches

3.1. Flat-jack and shear tests

Average results of two experimental tests (one for each wall in the x and y directions) were registered during the flat-jack and shear tests. Test locations for each church are shown in Figure 2.

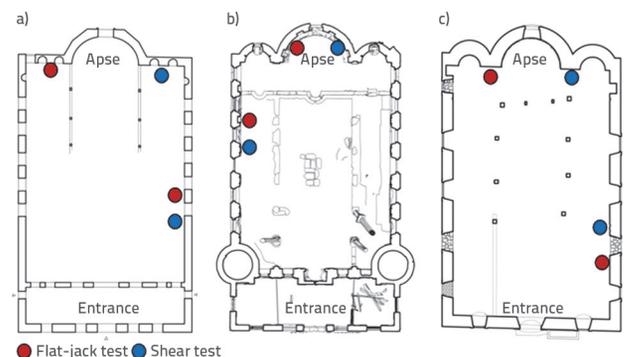


Figure 2. Flat-jack and shear test locations: a) Aydınpinar Church, b) Dereköy Church and c) Ballıpınar Church

The compressive stress, elastic modulus, and shear stress values were determined for masonry walls during these tests. A single flat-jack test method was used to determine

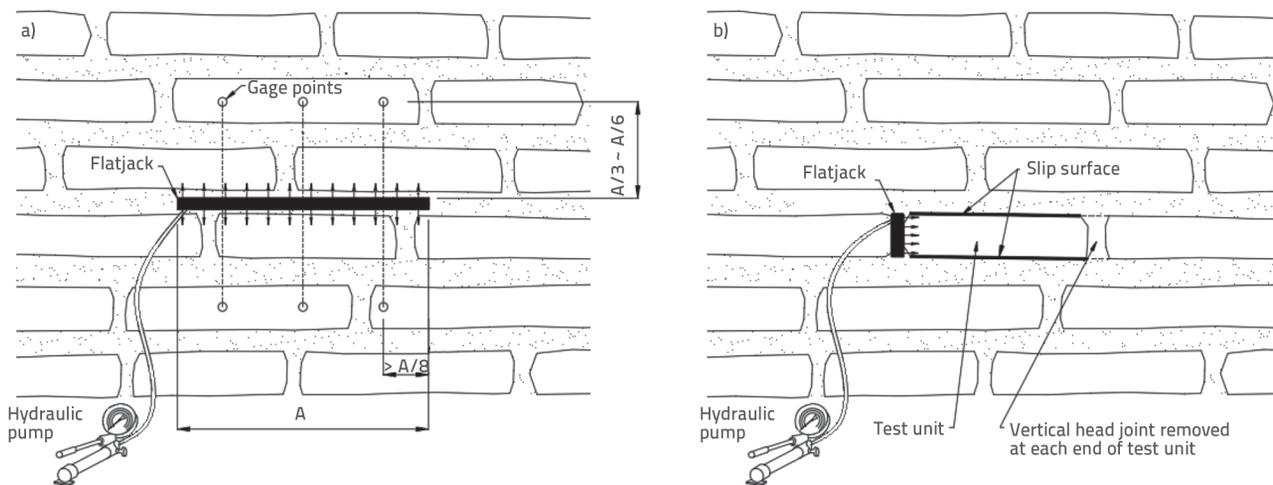


Figure 3. Flat-jack and shear test: a) Single flat-jack test setup b) Shear test setup

the compressive stress and elastic modulus of the walls. In this test method, a slot was cut in the mortar layer and a flat jack was placed inside the slot. The stress and displacement values of composite walls were continuously read while the hydraulic oil pressure pump (bar and psi) inflated the flatjack [23]. The method C was used according to ASTM C1531-09 in shear tests [24]. In this method, a small flat jack is horizontally inserted at one end of the test unit. Hydraulic oil pressure is applied until a crack appears or slip occurs.

A detailed presentation of the flat-jack and shear test setups, including the displacement measurement, flat-jack placement, and slip surfaces, is given in Figure 3. The calculation of compressive stress, elastic modulus, and shear stress is presented as follows in Eqs. (1) and (2).

Compressive stress exp. (1):

$$F_m = K_m \cdot K_a \cdot p \tag{1}$$

where K_m is a dimensionless constant related to the stiffness and geometrical properties of the flat-jack, K_a is the ratio of the area of the flat-jack to the area of the slot and p is the flat-jack pressure, psi or MPa.

The chord modulus at any point i , exp. (2):

$$E_{si} = \frac{f_{mi}}{\varepsilon_{mi}} \tag{2}$$

where f_{mi} is the stress at point i , and ε_{mi} is the strain at point i [23].

Elastic moduli of composite walls were calculated using end points of 0.05 and 0.33 chord modulus of elasticity values [25]. The stress-strain diagram of composite walls is presented in Figure 4.

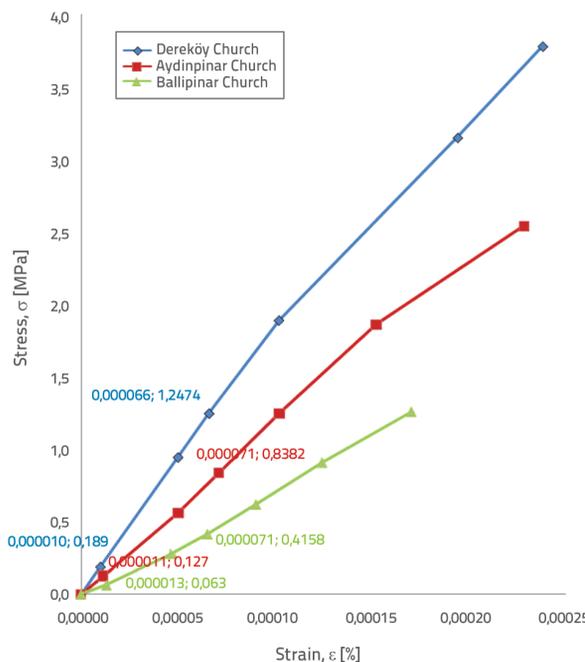


Figure 4. Stress-strain diagram of composite walls

The calculation of an average bed joint shear stress is given by equations (3) and (4).

Horizontal force:

$$P_h = K_m \cdot A_f \cdot p \tag{3}$$

where K_m is the dimensionless constant related to the stiffness and geometrical properties of the flatjack, A_f is the area of the flatjack and p is the flatjack pressure at crack initiation or slip.

Shear stress:

$$\tau = \frac{P_h}{A_j} \tag{4}$$

where P_h is the maximum force applied by the hydraulic pump and A_j is the gross area of the upper and lower bed joints [24]. Mechanical properties such as compressive stress, shear stress and elastic moduli of church walls calculated by in-situ tests are presented in Table 2. The plaster of the outer walls has fallen off and the open air conditions and moisture made the mortar weak. Therefore, the walls have significantly lost their mechanical properties.

Table 2. Mechanical properties of church walls

Churches	Compressive stress [MPa]	Shear stress [MPa]	Elastic modulus [GPa]
Aydınpınar Church	2.54	0.46	12
Dereköy Church	3.78	0.62	19
Ballıpınar Church	1.26	0.28	6

3.2. Structural health monitoring tests

The Operational Modal Analysis (OMA) test method was used to obtain dynamic parameters relevant for the studied churches. This non-destructive test method can be used to determine natural vibration frequencies, damping ratios, and mode shapes. This technique can be used to determine dynamic parameters of structures from output-only experimental data. The loads are environmental forces and the modal identification is based on responses only.

2400 mV/g highly sensitive accelerometers were used in the tests. Monoaxial accelerometers having 0.01-200 Hz bandwidths and $\pm 3g$ measurements range were used in this study. The Testbox 2010 data acquisition device with 8 channels was used in dynamic tests [26].

30-minute test periods were applied to ensure proper accuracy of the tests. During sensor placement, perpendicular

orientations were carefully checked for all church walls. For all churches, tests were performed both on the entire structure and on one of the individual walls in x direction. Accelerometers were placed approximately 1,5 metres under the elevation of the roof. The orientation of sensors and the test setup were the same for all churches, as shown in Figure 5 for Ballıpınar Church. Sensor placement and test equipment are shown in Figure 6.

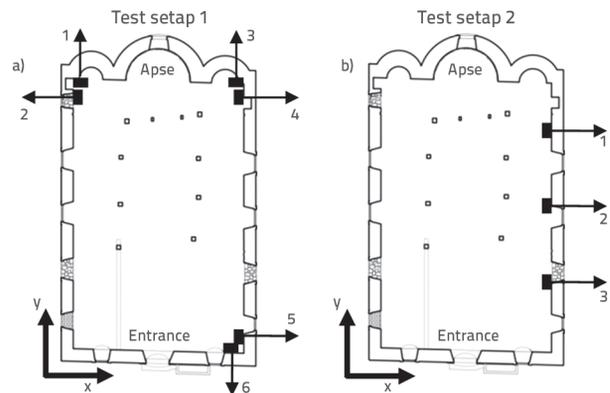


Figure 5. Plan view of sensor orientation for a) test setup 1, and b) test setup 2

The results for the test setup 1 were examined. It was observed that there were some difficulties in determining frequencies of the entire structure. The main problem was the absence of the roof and the loss of all horizontal connections between the walls. Therefore, it was seen that the walls do not act as a structure but rather as independent walls. The other problem was the non-symmetrical wall construction. The bending mode frequencies of the walls in x direction (north and south wall) were determined by OMA tests with good accuracy. The reason for this is thought to be the symmetrical wall construction and the sufficient test data gain from both walls. On the other hand, the bending mode frequencies of walls of the entrance part and the walls of the apse part in y direction (west and east walls) were determined by OMA tests with poor accuracy because



Figure 6. Vibration tests (Sensor placement and test equipment)

of the non-symmetrical construction techniques. Differently from walls of the entrance part, half-domes were used in the construction of the walls at the apse part. For these reasons, only the bending modes in x direction were considered, and the orthogonality between the first three bending modes was checked by the modal assurance criteria (MAC). The test setup 2 was also performed by placing three accelerometers in order to determine the first three bending modes of an individual wall of the churches in x direction. Very close frequencies between the test setup 1 and test setup 2 were observed for all churches. In addition, the first three bending mode shapes could be experimentally determined with greater certainty by the test setup 2 compared to the test setup 1. The singular values of the

spectral densities, the 3D and numerical presentation of MAC values, and the first three bending mode shapes in x direction obtained experimentally from test setups for Ballıpinar Church as an example belonging to test setups 1 and 2, are presented in figures 7 and 8.

4. Finite element analysis of churches

Natural frequencies and mode shapes of the churches were investigated by means of the Algor V20 finite element analysis (FEA) programme [27]. Brick, tetrahedra, wedge and pyramid elements, with three degrees of freedom at every node, were generated in the mesh of FEA models (Figure 9). In the finite

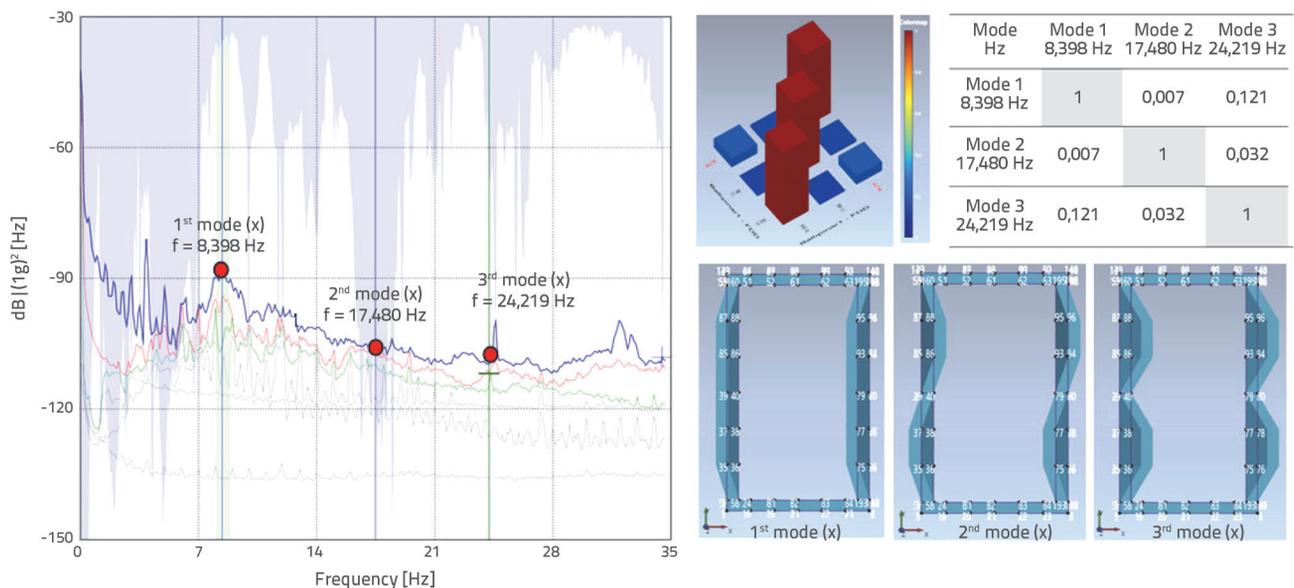


Figure 7. Singular values of spectral densities, 3D and numerical presentation of MAC values of Ballıpinar Church, and the first three bending modes obtained experimentally (Test setup 1)

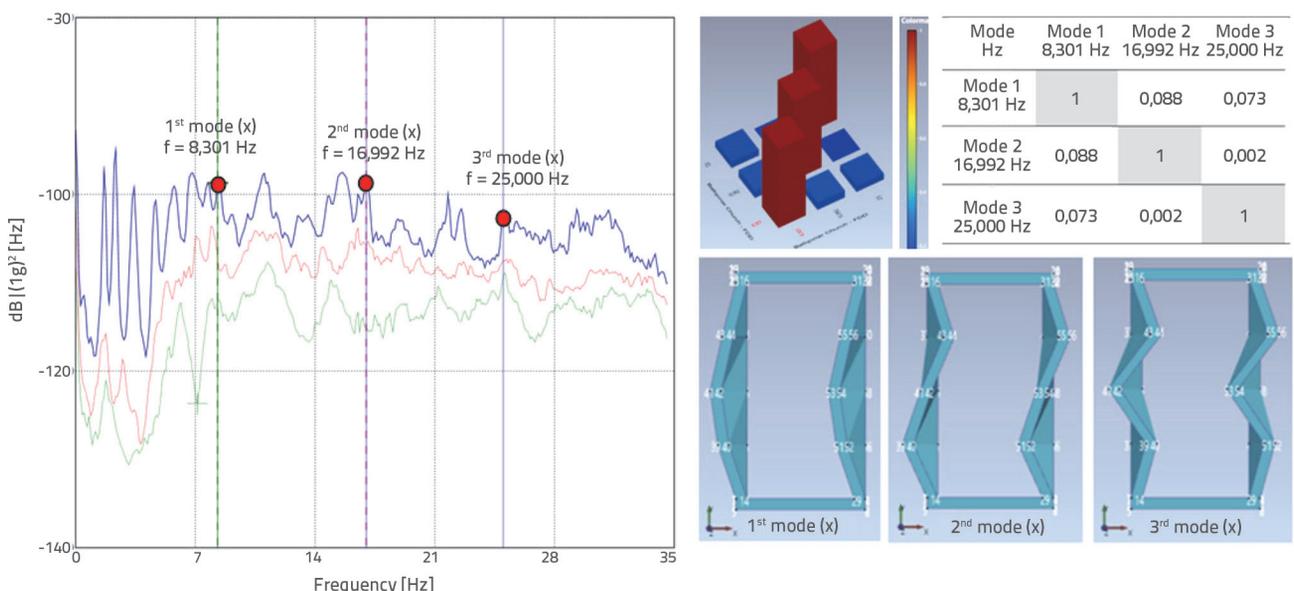


Figure 8. Singular values of spectral densities, 3D and numerical presentation of MAC values of Ballıpinar Church, and the first three bending modes obtained experimentally (Test setup 2)

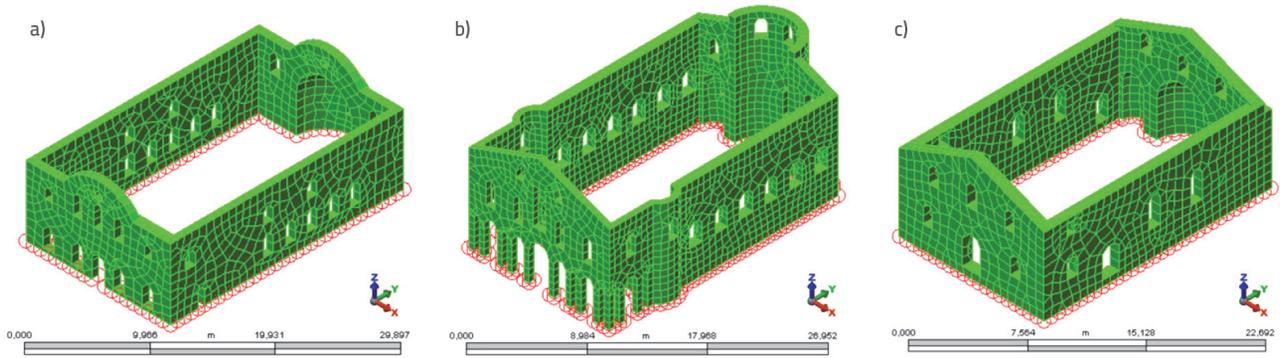


Figure 9. Finite element models for: a) Aydınpinar Church; b) Dereköy Church; c) Ballıpinar Church

Table 3. Material properties of churches after FEA calibration

Churches	Compressive stress [MPa]	Shear stress [MPa]	Elastic modulus [MPa]	Density [kN/m ³]
Aydınpinar Church	2.54	0.46	10	23
Dereköy Church	3.78	0.62	15	24
Ballıpinar Church	1.26	0.28	6	24

element analyses, 2927, 3007 and 5502 solid elements were used in Ballıpinar, Aydınpinar and Dereköy Churches, respectively. After the modal analysis, FEA models were calibrated so as to obtain the real behaviour of the structures. After calibration of finite element models, it was established that the elastic moduli obtained from test results are important for the accuracy of the models. The elastic modulus for Ballıpinar Church obtained by testing was 6 GPa and the value obtained by the FEA calibration was also 6 GPa (0 % error). The elastic modulus for Aydınpinar Church obtained by testing was 19 GPa and the value obtained by FEA calibration was 15 GPa (21 % error). The elastic modulus for Dereköy Church obtained by testing was 12 GPa and the value obtained by FEA calibration was 10 GPa (17 % error). The error between the test results and after FEA calibrations ranges from 0 to 21 percent. The Poisson’s ratio is taken as 0.16 for the masonry walls. Material properties of churches after FEA calibration are presented in Table 3.

The bending frequencies in x direction obtained by OMA tests were compared with the bending frequencies obtained from the finite element modal analysis for the calibration. The

frequencies of the first three bending modes of the walls in x direction (north and south wall) of Ballıpinar Church obtained from the OMA test setup 1 and the finite element analysis after the calibration is shown in an example given in Figure 10. In this figure, the bending frequencies obtained from the Finite Element Analysis are presented as FEA, while the frequencies from experimental study are presented as Exp.

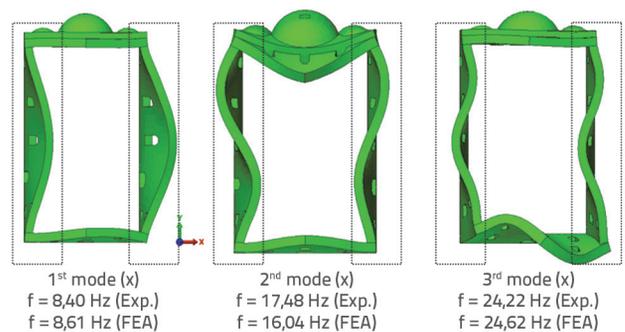


Figure 10. First three bending modes and frequencies of walls in x direction for Ballıpinar Church (experimental study versus finite element analysis)

Table 4. First three bending modes frequencies of church walls in x direction (experimental study versus Finite Element Analysis)

Mode number	Aydınpinar Church			Dereköy Church			Ballıpinar Church		
	Exp.	FEA	Err.	Exp.	FEA	Err.	Exp.	FEA	Err.
1 (x)	7.32	7.65	4 %	7.81	8.51	8 %	8.40	8.61	3 %
2 (x)	16.02	14.91	7 %	17.97	16.91	6 %	17.48	16.04	8 %
3 (x)	23.10	22.64	2 %	25.10	26.30	5 %	24.22	24.62	2 %

Exp.: Experimental, FEA: Finite Element Analysis, Err.: Error

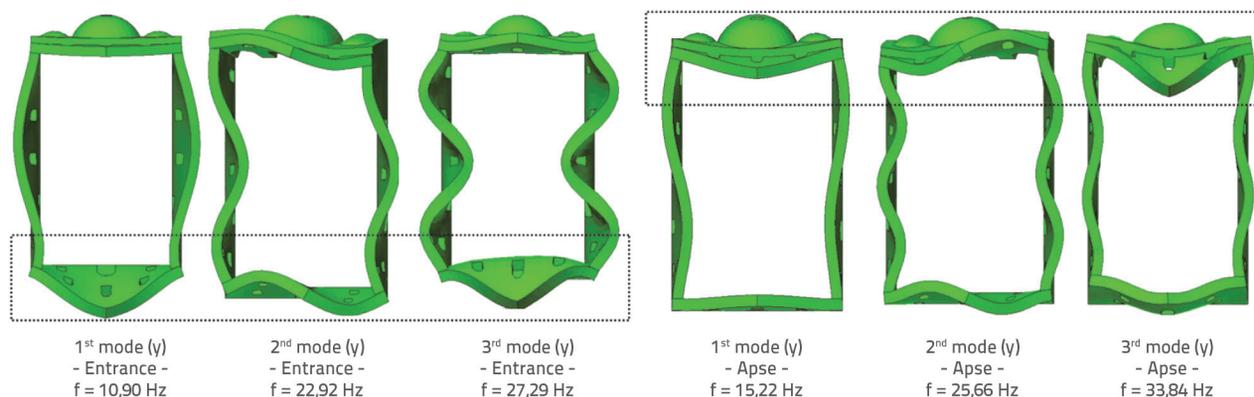


Figure 11. First three bending modes of Ballıpınar Church walls in y direction (entrance wall and apse wall)

Table 5. First three bending mode frequencies of church walls in y direction (entrance wall and apse wall)

Wall	Mode number	Frequency [Hz] Aydınpınar Church	Frequency [Hz] Dereköy Church	Frequency [Hz] Ballıpınar Church
Entrance wall	1 (y)	9.77	9.93	10.90
	2 (y)	19.83	21.49	22.92
	3 (y)	26.71	30.50	27.29
Apsse wall	1 (y)	15.65	15.84	15.22
	2 (y)	24.34	28.53	25.66
	3 (y)	34.57	35.12	33.84

Table 6. Mass participation factors for first three bending modes of studied churches in x and y directions

Churches	First mode		Second mode		Sum of first three modes	
	x	y	x	y	x	y
Ayıdınpınar Church	24.17	23.43	15.52	4.53	44.89	39.71
Dereköy Church	28.66	15.90	11.68	2.48	44.73	28.95
Ballıpınar Church	24.85	18.27	18.16	3.16	50.67	32.58

The mode frequencies obtained from experimental tests, mode frequencies obtained from finite element analysis, and the error between them, are presented in Table 4. The errors between the results ranged from a minimum of 2 % to a maximum of 8 %. The frequencies of the first three bending modes of walls in y direction (the wall of the entrance part -south wall, and the wall of the apse part - east wall) of Ballıpınar Church obtained from the finite element analysis after the calibration are shown as an example in Figure 11. It can be seen that the bending mode frequencies of the wall of the entrance and the wall of the apse in y direction differ from each other.

The first three bending mode frequencies of the wall of the entrance part and the wall of the apse part of churches in y direction were determined by the finite element analysis. The finite element analysis results can be seen in Table 5.

The mass participation factors of the first three bending modes of the studied churches in x and y directions are presented in Table 6. After the first three bending modes, it can be seen that the mass participation factors for x and y directions are below or close to 50 %. The mass participation factors increase to 95 % percent or over after 50 modes. This shows that at least 50 modes should be taken into account for the static and dynamic structural analysis of the structures.

5. Conclusion

Dynamic behaviour of three 19th century churches in the cities of Bursa and Balıkesir were investigated in this study. The flat-jack and shear tests were performed to determine mechanical behaviour of the walls. When the vibration test results were

compared with the finite element analysis, it was established that the mechanical properties were determined with a high level of accuracy. The maximum difference between the vibration tests and the finite element analysis ranged between 0 and 21 percent.

The vibration tests were conducted in order to determine dynamic characteristics of the structures. Unfortunately, some difficulties were encountered when determining frequencies of the entire structure. The first one was that the walls did not act as a structure but as independent walls due to the loss of the roof and horizontal connections between the walls. The walls in y direction had different vibration frequencies. Unlike the wall of the entrance part, half-domes were used in the construction of the walls of the apse part. On the other hand, the bending mode frequencies of the walls in x direction (south and north walls) were determined by OMA tests with great accuracy. The reason for this is thought to be the symmetrical wall construction and the sufficient test data collected from both two walls. In addition, the first three bending mode frequencies and mode shapes for one of the individual walls in x direction of the churches were obtained from another test setup. The orthogonality between the first three bending

modes for both test setups in x direction was checked using the modal assurance criteria (MAC). It was observed in both tests for all churches that the frequencies are very close to each other. After calibration of material properties, the errors between test results and the finite element analyses ranged from a minimum of 2 % to a maximum of 8 %.

The identification of structural and material behaviour of structures is important for the conservation studies. Therefore, it is necessary to diagnose the current condition of structures in detail so as to be able to properly evaluate their safety level. It is clear that the flat-jack, shear and vibration tests have an important role in correct determination of the properties and dynamic behaviour of these structures. In the case of the studied churches that are currently reduced to load bearing walls (outer walls) only, a complete analysis should involve the testing and evaluation of both the individual walls and the entire structure using finite element models.

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